

Visual Computing

Image Based Computer Graphics

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Arts Sciences
TH Köln

Andreas Karge

Advanced Media Institute

WS 2020/2021

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Me and My Motivation

- Photographer
- TH Köln: Photo-Engineer and Media Informatics
- GMG: Colour Management for Printing Industry
- HdM Stuttgart: Colour Management for Movie Industry
- University of Tübingen: PhD student, Physical Based Colour Image Processing



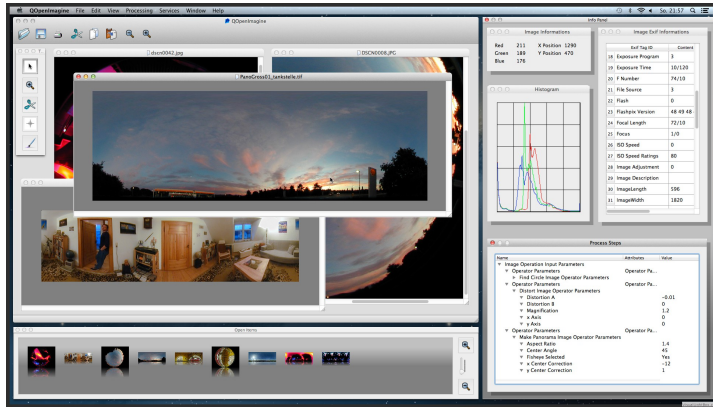
My Family and Me

Learn more about digital image processing to drive research and applications in:

- Photography
- Cinematography
- Machine Vision

Photography Example: QOpenImagine

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3D Sphere Mapping...

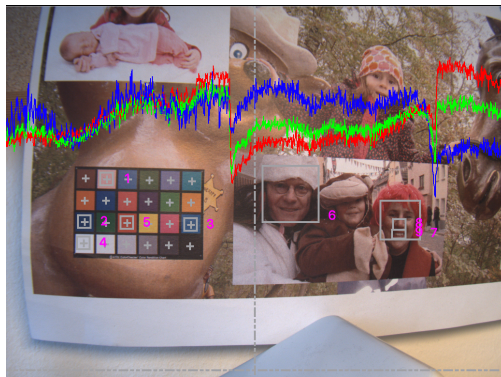
Movie Example: OpenFilmTools

Movie



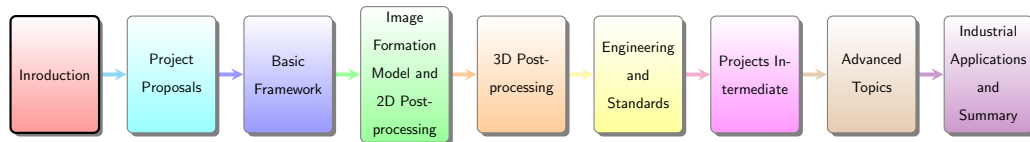
Colour management which uses spectral data based camera characterisation.

Machine Vision Example: QOpenCapture



Pattern and Face Recognition

V1 Introduction to Image Based Computer Graphics



Content I

1 V1 Introduction to Image Based Computer Graphics

Focus

- Get a first overview in image processing
- Learn common naming conventions
- Know the interdisciplinary context
- Have a lecture guideline

Naming, Classification

Image Based Computer Graphics

- Visual Computing: discipline in computer science focused on visualization
- Image base computer graphic is a part of visual computing and is the superset of
 - computer graphics
 - image processing
 - computer vision
- It is focused on image processing of real world scenes.

Challenges and Applications

- Challenges: acquisition, processing, analysis and rendering of still images and image sequences
- Applications: industrial quality control, medical image processing and visualization, remote sensing, automotive, robotics, multimedia systems, virtual heritage, special effects in movies and television

Methods and Skills

- Prerequisite: understand the image formation of real world scenes in human and camera observers
- Methods: mathematical and algorithmic methods dealing with images: image formats, filtering methods, color models, image comparison metrics
- Method Implementation Skills: programming methods on CPU/GPU, huge data resource management, real time processing

Motivation in Detail

Motivation in Detail

Reconstruct the captured real World by image preprocessing:

- Preprocessing: scene reconstruction, remove all artefacts from image capturing
- Color processing: "Camera Characterization", i.e. scene color reconstruction

Motivation in Detail

Enhance/Extract image information by image postprocessing:

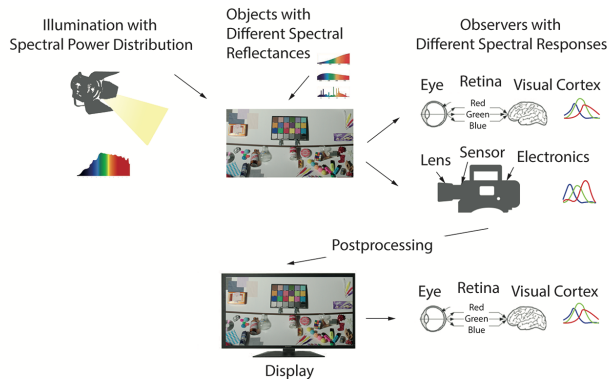
Postprocessing:

- 2D: Filtering, feature extraction, segmentation, classification
- 3D: Reconstruction
- Artistic modification (effects not provable by metric/measurement systems)
- Display Characterization: scene color reconstruction for output presentation
- Encoding

General Image Processing Pipeline

Real World Image Processing Pipeline

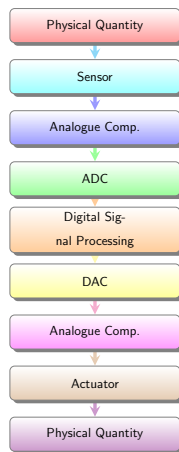
Before we process images, we should understand the complete image formation...



Vision and Image Processing

Digital Image Processing System

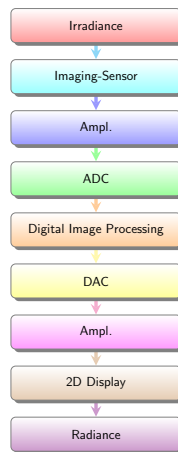
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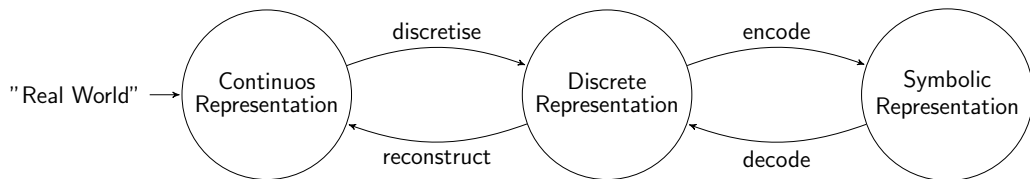
Digital Signal
Processing (left)



Digital Image
Processing (right)



Digital Image Processing Cycles



Understanding the Pipeline: Related Sciences, Engineering, and Information Sciences

Understanding the Pipeline

Related Sciences Topics

- Mathematic: Geometry, Algebra, Numeric (Solver/PCA)
- Physics: Radiometry, Optics
- Biology: Anatomy of Eye (Humans/Animals)
- Psychophysics: Photometry, ("lower") Colorimetry: Stimulus
- Neuro/Cognitive Science: ("higher") Colorimetry

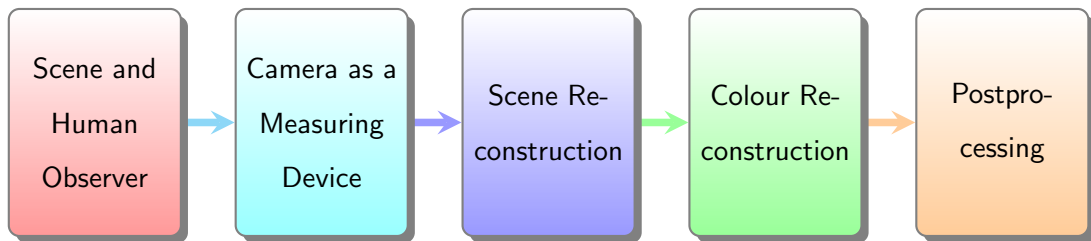
Understanding the Pipeline

Related Engineering and Information Sciences Topics

- Signal Processing
- Machine Learning
- Software Engineering
- Illumination
- Material Science: Lenses, Semiconductor Imaging Sensors

Focus of Image Processing in WS2020/2021

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Preprocessing: scene signal and colour reconstruction

Postprocessing: 2D filter, feature extraction, and 3D reconstruction

if time left: output colour rendering, e.g. display colour management

Practical Things

Communication, Lecture Format, Project Groups

- Ilias Platform, [BCG - Bildbasierte Computergrafik](#)
- Contact: [andreas.karge\(at\)th-koeln.de](mailto:andreas.karge(at)th-koeln.de)
- Skype: Andreas Karge
- Lecture 2x45 min, 15 min break, optional project group talks afterwards
- Project group size: 1-3

Timeline I

1. Semesterplan Informatik WS 2020/2021 - Masterstudiengänge																				
Woche	19.10. - 23.10.	26.10. - 30.10.	02.11. - 06.11.	09.11. - 13.11.	16.11. - 20.11.	23.11. - 27.11.	30.11. - 04.12.	07.12. - 11.12.	14.12. - 18.12.	21.12. - 25.12.	28.12. - 01.01.	04.01. - 08.01.	11.01. - 15.01.	18.01. - 22.01.	25.01. - 29.01.	01.02. - 05.02.	08.02. - 12.02.	15.02. - 19.02.	22.02. - 26.02.	März 2020
optionale Blockveranstaltungen (teils vor Ort)										Feiertage / vorlesungsfrei										
09.11.2020 - 13.11.2020 25.01.2021 - 29.01.2021 15.02.2021 - 26.02.2021										Weihnachten/Jahreswechsel 21.12.2020 - 01.01.2021 Rosenmontag 15.02.2021										
Prüfungszeiträume										Vorlesungszeiten (digital)										
19.10.2020 - 30.10.2020 01.03.2021 - 31.03.2021										entsprechend Ihrem Veranstaltungsplan										

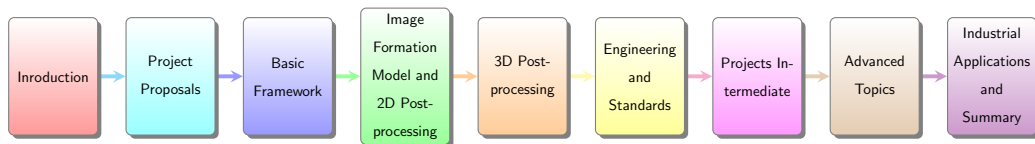
Timetable

Timeline II

- Today: Introduction
- Next Week: Projects 1 day, 13-11-2020, 1:00 pm ...
- 5+3 lectures
- Projects: intermediate presentations, 25., 26., 27. and 28. January 2021, 09:00 am (s.t.) -1:00 pm, 29. optional depending on groups sizes
- 2 lectures
- Live demonstration/talk, to be clarified: 1 day each or both together
- Projects: Final presentations, to be clarified: format

Lecture Guide

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... and afterwards project presentations

Programming Skills

- Octave/Matlab
- Python, Python Notebook, APIs: NumPy, SciPy, PyTorch
- C++, APIs: Eigen, OpenCV
- UI/nonUI executable, Qt
- Repository: GitHub, ...
- Documentation: Latex

Recommended Books I

- R.C. Gonzalez and R.E. Woods. *Digital Image Processing*. Pearson Education, 2011. ISBN: 9780133002324. URL:
<https://books.google.de/books?id=MaYuAAAAQBAJ>
- B. Jähne. *Digital Image Processing*. Springer Berlin Heidelberg, 2013. ISBN: 9783662034774. URL:
https://books.google.de/books?id=%5C_SPyCAAAQBAJ

Recommended Books II

- B. Jähne et al. *Technische Bildverarbeitung — Maschinelles Sehen*. Springer Berlin Heidelberg, 2013. ISBN: 9783642614033. URL: <https://books.google.de/books?id=b98nBgAAQBAJ>
- Marcelo Bertalmio. *Image processing for cinema*. CRC Press, 2014
- Robert Schaback and Holger Wendland. *Numerische Mathematik*. Springer-Verlag, 2006
- Sijia Wen et al. “Joint Chromatic and Polarimetric Demosaicing via Sparse Coding”. In: *arXiv preprint arXiv:1912.07308* (2019)

Recommended Books III

- Alfred Nischwitz, Max Fischer, and Peter Haberäcker. *Computergrafik und Bildverarbeitung*. Springer, 2007
- Reinhard Klette. *Handbuch der Operatoren für die Bildbearbeitung: Bildtransformationen für die digitale Bildverarbeitung*. Springer-Verlag, 2013
- E. Hecht. *Optics*. Pearson, 2012. ISBN: 9788131718070. URL: <https://books.google.de/books?id=wcMWpBMMzIkC>

Summary

V1 Introduction to Image Based Computer Graphics

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Summary

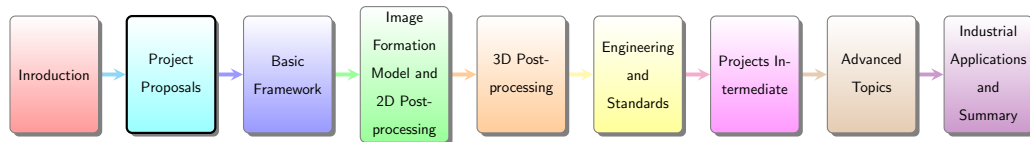
- Industrial imaging applications drives image based computer vision.
- Automatic image processing instead of manual adjustments are required.
- Restrictions are defined by lighting, objects, cameras and human observer.

Take Aways

If you design an image processing algorithm, define the whole image processing system, ask for sharp quantitative requirements in your application and break down your system.

Day 2

P1 Full Day: Project Proposals



Content I

- 2 P1 Full Day: Project Proposals
 - Common Project Notes
 - Proposals from V1-8
 - Own Proposals

Focus

Focus

Projects reflecting lecture units

- Project requirements
- Proposals and own ideas

Common Project Notes

Time Line for Kick Off

- At the end of proposal presentation, a 15 min break follows
- Afterwards we have a talk about your ideas and spontaneous groups/projects can be build
- Till 15th of December all groups should send a project expose
- For 2D filtering, segmentation and classification, and for 3D image processing, the lectures will be held afterwards till mid of January but datasets can be created even before related lecture

Project Types

... two possible kinds of project realization are offered

- With own camera hardware: define your image capture experiment in photography/movie/machine vision context
- Without own camera hardware: samples can be given or might be found

Outcomes

- GitHub Repository of code
- Documentation == paper
- Poster presentation

Paper/Poster Parts

Combination of academic research and engineering scopes

- Research of state of the art algorithms/tools
- (Experimental setup)/Algorithm description
- Defend the choice of your algorithm
- Quantitative/(qualitative) image evaluation (ground truth)
- Evaluation against other implementations/unit tests
- Algorithm evaluation for memory/CPU/GPU usage (benchmark)

Comparison

- Own ground truth
- Use reference implementations to compare: OpenCV.../Photoshop/Nuke

Project Realization Framework

Soft Skills

- Take care of (conference) requirements
- Story telling: even in academic research, you have to tell a story
- Opener: visualization image
- No Go: unproven phrases
- Time budget, prioritization of tasks

Paper/Poster Sample

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- *Appetizer...*
- *Poster...*
- *Paper...*

Possible Frameworks

- Octave/Matlab
- Python, Python Notebook, APIs: NumPy, SciPy, PyTorch
- C++, APIs: Eigen, OpenCV
- Repository: GitHub
- Documentation: Latex, optional you can use a conference template (IEEE, SMPTE...)
- Project management: Scrum/agantty

Image Files

- Input raw or TIFF
- Be aware of colour domain, if non linear, linearise before
- Use raw mosaic from dcraw
- Use simple demosaic from `"dcraw -r 1 1 1 1 -M -o 0 -4 -T -h "`
- Work with 16bit int or 32 bit float inside
- If you use color images be aware of color domain documentation
- Output TIFF

Interfaces

Because projects code might be used together in TH-Köln framework pipeline in future, we should define together:

- Input images with supported format TIFF, n-channels/16bit int/32 bit float per channel
- Output Images with supported format as above
- Constraints configuration file, json/xml
- Common image interface for in memory images
- Common image processing pipeline interface

Interface Proposal

```
class IPixmap
{
public:
    static IPixmap* Create(const std::wstring& imageFileName, bool tryAll = false, bool addAlpha = false);
    static IPixmap* Create(size_t width, size_t height, size_t nOfPixelElements,
                           size_t sampleType, size_t fillBytes,
                           const unsigned char*& buffer,
                           const std::string& signature);

    static bool Save(const std::wstring& imageFileName, const std::shared_ptr<IPixmap>& pixmap);
};
} //end namespace
```

Listing 1: Image Entity

Interface Proposal

```
namespace TH-Cologne::AMI::Image {  
class IPixmap  
{  
public:  
    virtual void* GetStartPointer() const = 0;  
    virtual void* GetLineStartPointer(int line) const = 0;  
  
    virtual size_t GetWidth() const = 0;  
    virtual size_t GetHeight() const = 0;  
  
    virtual size_t GetNumberOfPixelElements() const = 0;  
    virtual size_t GetSampleType() const = 0;  
    virtual size_t GetLinePadding() const = 0;  
};  
} //end namespace
```

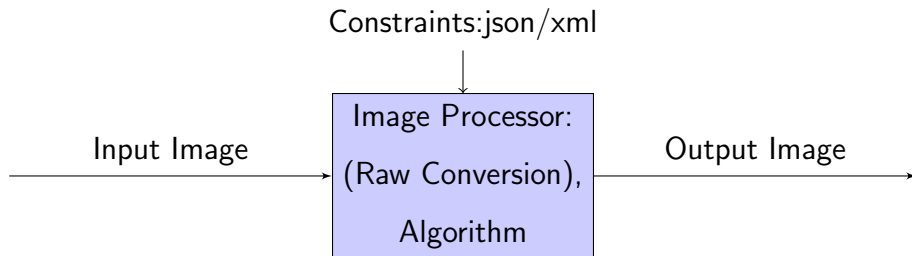
Listing 2: Image Entity

Interface Pipeline Proposal

```
namespace TH-Cologne::AMI::ImageProcessing {  
  
class IImageProcessingPipeline  
{  
public:  
    virtual void Init(string configFile) = 0;  
  
    virtual void Process(std::shared_ptr<AK::ImageData::IPixmap>& inputHostImage) = 0;  
  
    virtual void Process(GLuint& outputGPUImage) = 0;  
};  
  
} //end namespace
```

Listing 3: Image Processing Subsystem

Project Software Module



Think about parallel computing, and dividing in sub-algorithms

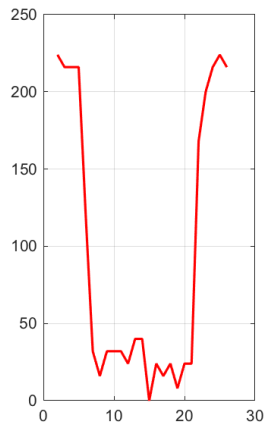
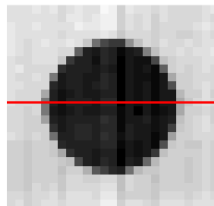
Proposals from V1-8

Object Distance, Line Distribution, Impulse Answer

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Line Profiles

- Iterative Autofocus Simulation
- Add Gaussian Blur to simulate out of focus

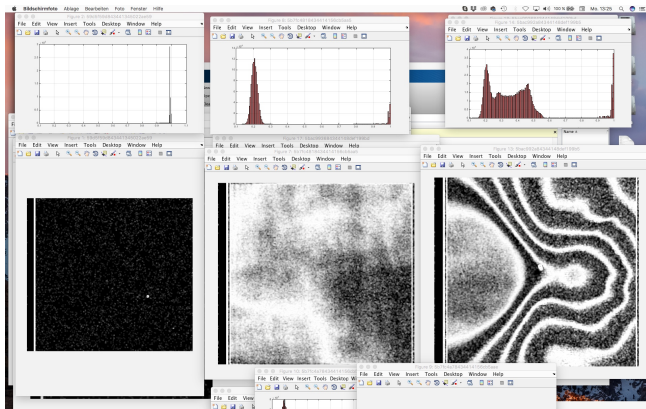


Noise Simulation and Removal

- Add noise to defined samples
- Median/Average filter
- Wiener filter (experiment: estimate impulse response)
- THIKONOV regularization

Noise Simulation and Removal

Brightest star on sky, a "Sirius" real world sample



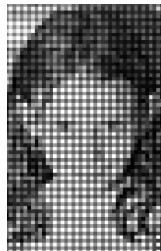
Demosaicing

(dc)raw to full rgb image

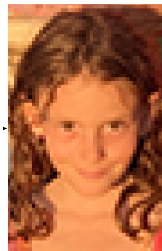
- Algorithm comparison:
nearest neighbour,
bilinear...
- Advanced: total variation

Samples

- Simulate by subsampling
real world images



raw to rgb



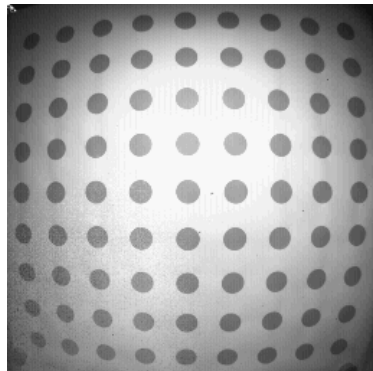
Geometric Calibration of Distortion

Lens calibration

- Test chart, rectangular vs. circle pattern
- Inner orientation - center, rectification
- Interactive/automatic point selection

Samples

- Simulate pattern distortion
- Estimate required resolution
- Real world



Geometric Calibration of Distortion

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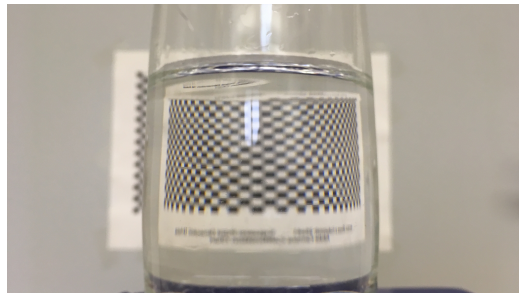
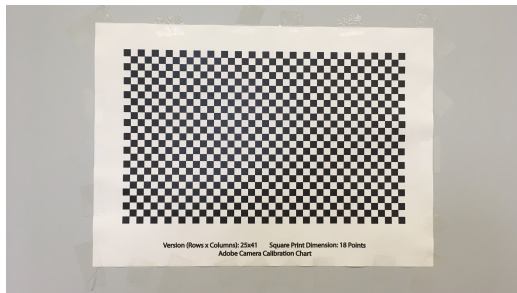


Chart Detection

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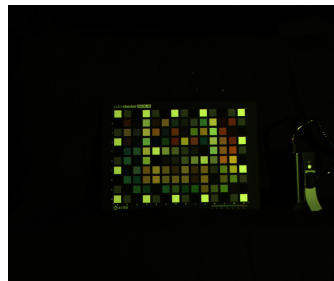


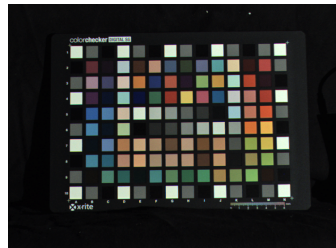
Chart Detection

- Generic patch detection for displaced chart
- ColorChecker, ColorChecker Digital SG and generic rectangular patch layout
- Two charts or more
- Matlab/OpenCV base exists

Spectral/Tristimulus based Color Profile Creation

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Calculate/Evaluate colour profiles



Spectral/Tristimulus based Color Profile Creation

Colour Transformations

- CIE-XYZ, ΔE , ΔE_{2000} and CIECam, linear/non linear optimization
- Convergence for increased number of patches
- -python notebook exists
- Write an ICC file (repo exists)
- Compare different spectral resolutions

Spectral/Tristimulus based Color Profile Creation

Samples

- Sample illumination data
- Sample camera response data
- Cross validation of ColorChecker complete/subset created profiles profiled color patches of ColorChecker ColorChecker SG

Blending of Color Profiles

Chromatic Adaptation

- Blending matrix
coefficients
- Can be done in Nuke for
movie production

Evaluation

- Qualitative observer
experiment

Coins

Count the money in your wallet



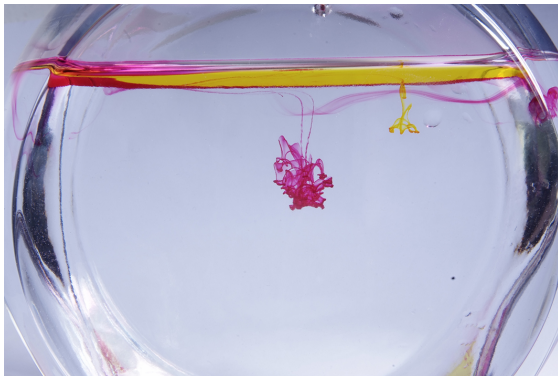
Under diffuse illumination



Under directional illumination

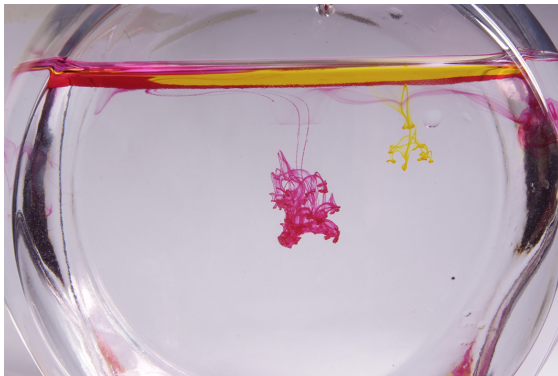
Estimate Velocity/Acceleration

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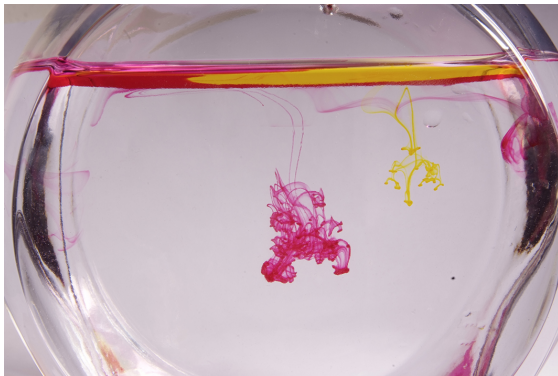
Estimate Velocity/Acceleration

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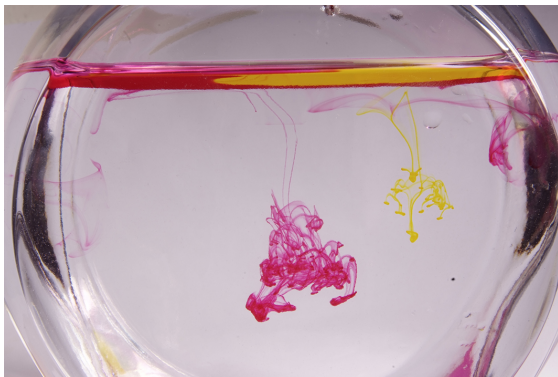
Estimate Velocity/Acceleration

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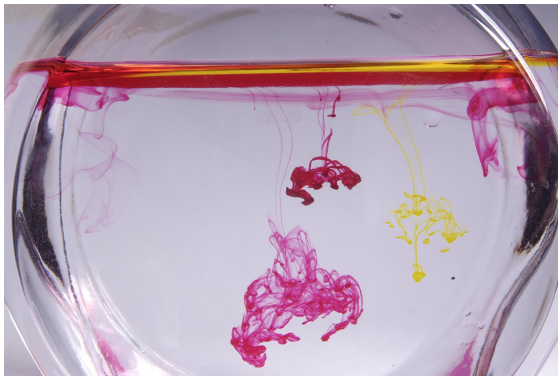
Estimate Velocity/Acceleration

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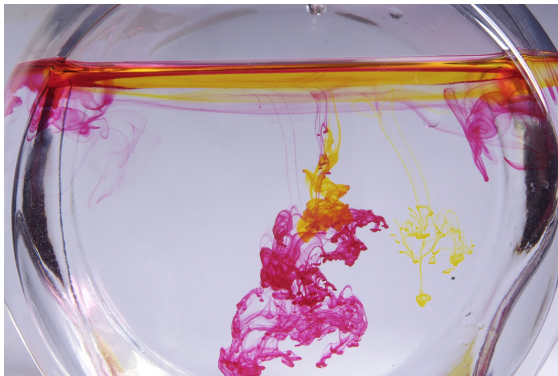
Estimate Velocity/Acceleration

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Estimate Velocity/Acceleration

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Face Detection

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Face Detection

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Face Detection

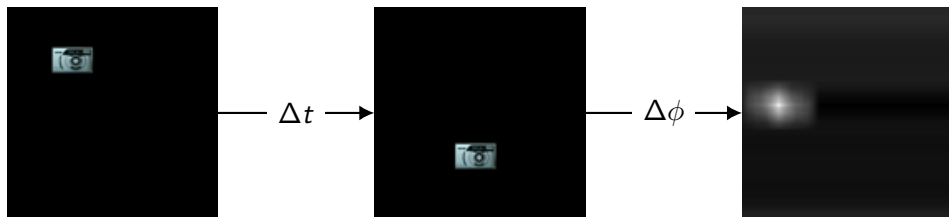
- A Python based repository including Notebook UI exists.
- Two algorithm parts: Classic/Machine Learning
- At least first one must be optimized.

Mean Face of our Class

- Find a mean face (Principal Points, PCA)
- Blend into other faces (Morphing)

Disparity Estimation

Spatial Domain Base and Phase Correlation



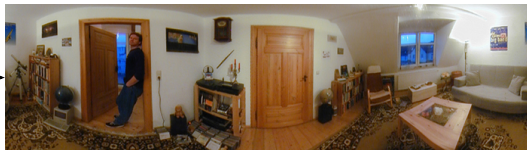
Sphere Mapping

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Fish Eye/Spherical Mirror



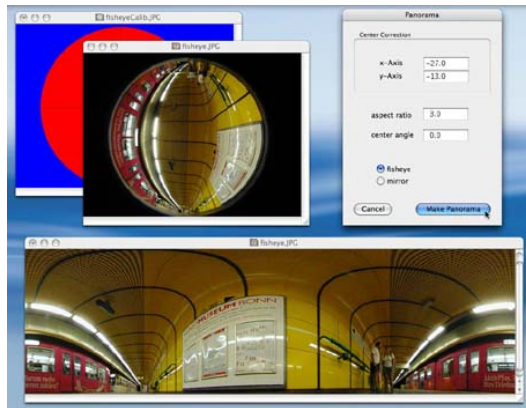
— pol to cartesian →



Sphere Mapping

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Fish Eye/Spherical Mirror



Line Intensity to Spectrum

Spectroscopy

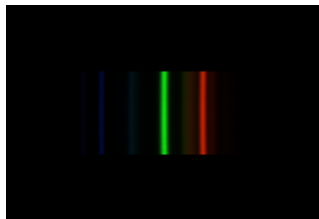
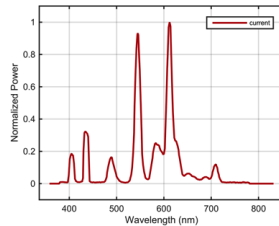


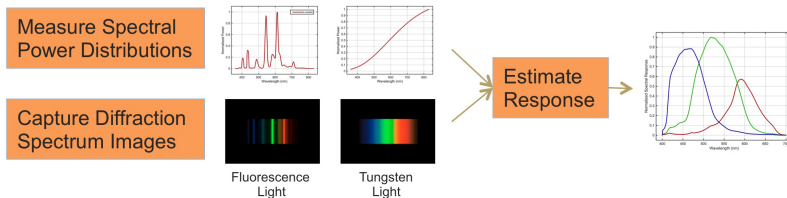
image to SPD →



Sample Image data exists...

Line Intensity to Spectrum

Camera Characterization



Sample Image data exists...

3D Reconstruction

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Movie



Complete Roundtrip

3D Reconstruction

Discontinuous sequence, Base/Distance 1 m

Interpolate intermediate position/Compare to middle origin



3D Reconstruction

Discontinuous sequence, Base/Distance 1 m

Interpolate intermediate position/Compare to middle origin



3D Reconstruction

Discontinuous sequence, Base/Distance 1 m

Interpolate intermediate position/Compare to middle origin



Find and Match Points/Lines

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Camera shake/motion blur influence

Find and Match Points/Lines

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Camera shake/motion blur influence

Find and Match Points/Lines

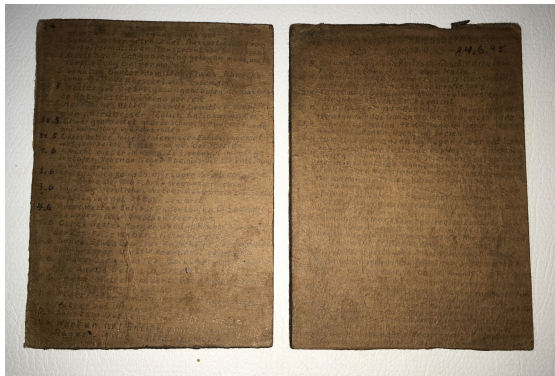
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Camera shake/motion blur influence

Reconstruct Bleached Text

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Cultural heritage: old bleached documents

Pinhole Camera

- Build your own pinhole camera
- Compare transfer function and images: pinhole/lens
- e.g. [look here...](#)

Own Proposals

Own Proposals

- Should be related to lecture sections
- Requirements must be specified
- If you need samples, please ask

Summary

Timeline

- Expose
 - Goal
 - Milestones
 - Tools/Infrastructure
 - Tasks per person
- Intermediate state presentation: group wise, slides
- Final presentation: all together, slides/poster/paper

P1 Full Day: Project Proposals

Summary

- Project requirements
- Common interfaces
- Proposals

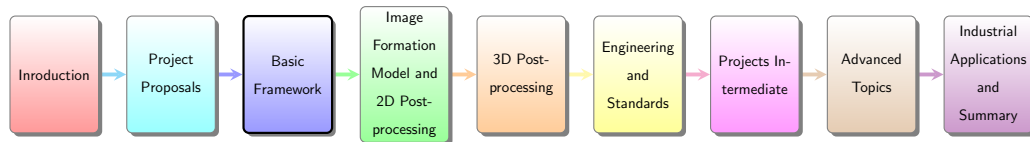
Take Aways

Defend your selected algorithm and implementation by quantitative evaluation.

lets have a break and talk afterwards...

Day 3

V2 Mathematical Framework for Digital Image Processing



Content I

- 3 V2 Mathematical Framework for Digital Image Processing
 - Digital Image Representation
 - Analysis Aspects
 - Image Descriptive Statistics
 - Image Distance Metric
 - Image Processing
 - Neighbourhood Operations

Content II

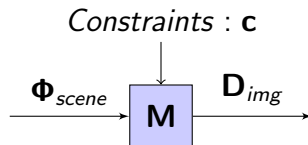
- 2D Image Reshaped in 1D Vector Notation

Focus

- Know image representations
- Formulate abstract image processing
- Learn software engineering aspects

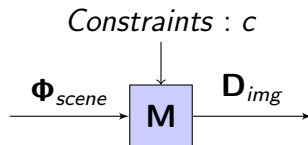
Digital Image Representation

Image Formation of the Real World



- **M**: Image Formation model
- Φ_{scene} : Surface radiances ϕ
- $\mathbf{D}_{img}$: Set of image intensities, e.g. pixel values $\mathbf{d}$
- $\mathbf{M}(\mathbf{c}) : \mathbb{R}^{ix \times jm} \rightarrow \mathbf{D}^{ix \times jn}, \phi \mapsto \mathbf{d}$
- $\mathbf{r} = (i, j)^T$ Sensor plane coordinates, $\mathbf{D}$ discrete numbers domain

System View



3 Aspects

α) Reconstruction: The original (scene) signal can be reconstructed from the captured image, if M and output is known by: β) Calibration: M can be estimated, if input and output is known. γ) Simulation: Finally if one knows input and M , the output can be simulated.

Digital Image Examples

$$\mathbf{D}_{img} = \begin{pmatrix} i_{0,0} & \cdots \\ \vdots & \ddots \end{pmatrix}, \text{ grayscale}$$

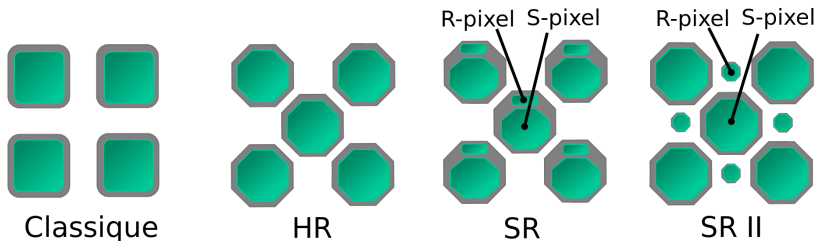
$$\mathbf{D}_{img,raw} = \begin{pmatrix} \begin{pmatrix} r_{0,0} & g_{0,1} \\ g_{1,0} & b_{1,1} \end{pmatrix} & \cdots \\ \vdots & \ddots \end{pmatrix}, \text{ raw sensor data, mosaicing with Bayer pattern}$$

$$\mathbf{D}_{img,interleaved} = \begin{pmatrix} \begin{pmatrix} r_{0,0} & g_{0,0} & b_{0,0} \end{pmatrix} & \cdots \\ \vdots & \ddots \end{pmatrix}, \text{ reconstructed full rgb pixels}$$

$$\mathbf{D}_{img,transp} = \begin{pmatrix} \begin{pmatrix} r_{0,0} & g_{0,0} & b_{0,0} & \alpha_{0,0} \end{pmatrix} & \cdots \\ \vdots & \ddots \end{pmatrix}, \text{ gpu image, e.g. in opengl, opencl, vulkan}$$

Spatial Domain

- Sensors might have no regular grid layout, i.e. in general the indices represent a look up table to sensor coordinates,
- Quadratic/rectangular/honeycomb/superCCD patterns



©Arachanox

Discrete Sampling and Signal Domain

- Sensors might not give the full sample vector
- E.g. only one channel sample exists from final RGB

Discrete Sampling and Signal Domain

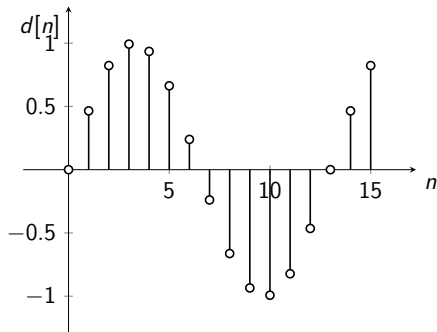
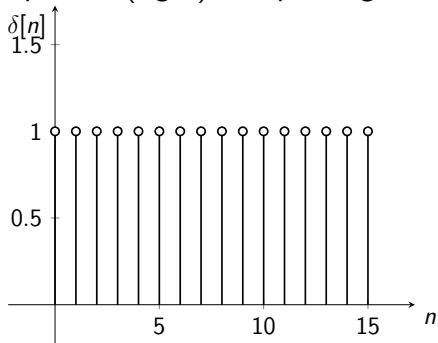
- Signal domain of stored image
- Not whole unsigned byte/int... domain is used
- (IEEE)float also can represent a minimum value which is negative
- Maximum value for "well exposed" images is below the saturation value
- Data must be normalised somehow, e.g. $(-0.01, 0.7)$ to $[0, 1.0)$

Artefacts

- Discrete numbers: numeric artefacts
- Sensor frame: boxing/bounding of real world, if algorithm requires neighbourhood values, fill with reasonable values

Sampling Theorem

NYQUIST/SHANNON sampling theorem, DIRAC delta function, (left) Unit step sequence, (right) Sampled signal



Artefacts from Sampling Theorem

- NYQUIST frequency: limited geometrical resolution, beats
- Quadratic pixels has lower diagonal resolution
- BAYER pattern has two times better resolution for green channel

Memory Layout Considerations

- Start
- Padding
- Interleaved/plane
- Alpha/dummy channel
- The 10/10/10 bit hack in OpenCL/Vulkan

Symbolic Presentation

- Code values must not be use n times bytes
- Encoding
- Persistence

From Spatial to Frequency Domain

An image signal can be reinterpreted as the superposition of weighted frequencies (FOURIER 1822)

$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \exp(-i2\pi(ux/M + vy/N)),$$

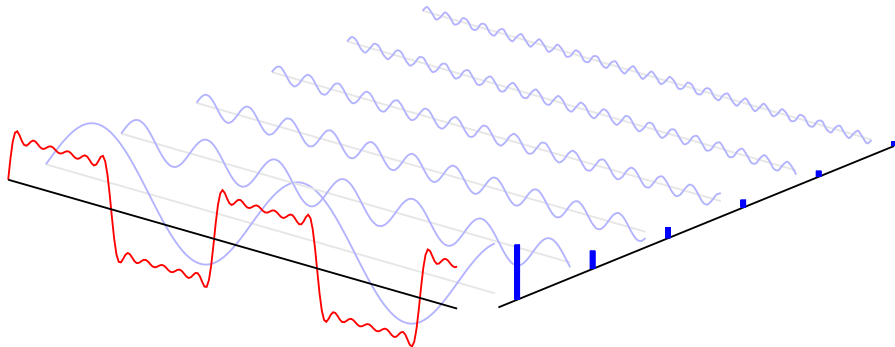
$$u = 0, 1, \dots, M-1, v = 0, 1, \dots, N-1$$

$$f(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \exp(i2\pi(ux/M + vy/N)),$$

$$x = 0, 1, \dots, M-1, y = 0, 1, \dots, N-1$$

From Spatial to Frequency Domain

Example with removed DC bias



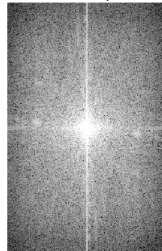
From Spatial to Frequency Domain Sample

- Shows image information, but also...
- ...bounding box artefacts
- Amplitude diagram: magnitude of frequency components

Image



Absolute Amplitude



Note: the FOURIER image can be created physically by an optical " $2f$ " setup

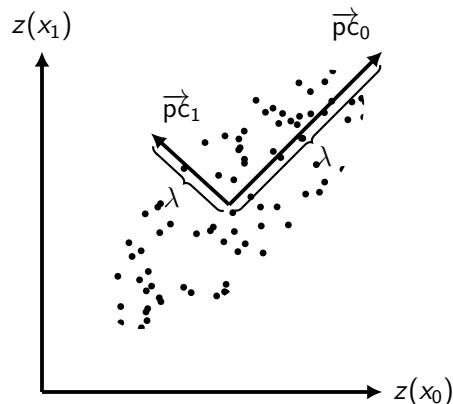
Other Unitary Transformations

In image processing FOURIER transform has a direct relation to optics, but also others can be used:

- Cosine term limited FT: DCT (JPEG)
- Generalized FT: Hadamard
- Set of other orthonormal functions: Wavelet (JPEG2000)

Principal Component Analysis

- Based on principal axis theorem
- Find a transformation for a given vector, whereas the resulting vector has an decreasing variance starting from the first element



Principal Component Analysis

Find a transformation for a given vector, whereas the resulting vector has an decreasing variance starting from the first element.



Analysis Aspects

Integration

Images can be (numerical) integrated, it gives the (normalized) power of an image

$$E_n = \frac{1}{xy} \sum_{x_i=0}^{x-1} \sum_{y_j=0}^{y-1} (d_n(x_i, y_j))$$

Note: light sensitive receptors integrate the squared amplitude of the electromagnetic field over time, or as another view they just count the photons

Differentiation

Images can be numerical differentiated in each channel. Important are the first order derivative: gradient ∇ and second order: divergence ∇^2

$\nabla d = (\partial d / \partial x, \partial d / \partial y)^T = (g_x, g_y)^T$ which can be approximated by the discrete differences Derived parameters:

- Direction $\tan^{-1}(g_y/g_x)$
- Magnitude $\sqrt{(g_y^2 + g_x^2)}$

Different implementations will be discussed in 2D Filter section.

Differentiation Sample

Magnitude/Direction of First Derivative



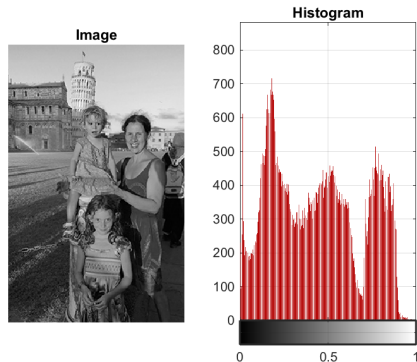
∇D



Image Descriptive Statistics

Histogram

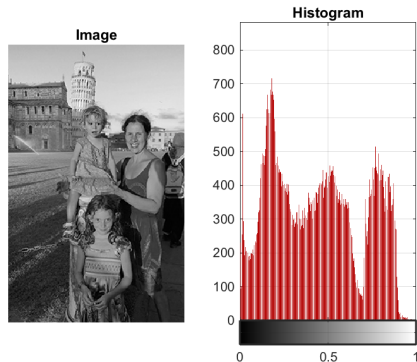
It represents the (normalized) probability $P(d)/P_{norm}(d)$ of occurrence of a code value in a channel. It is Poisson distributed.



Histogram

It helps to evaluate

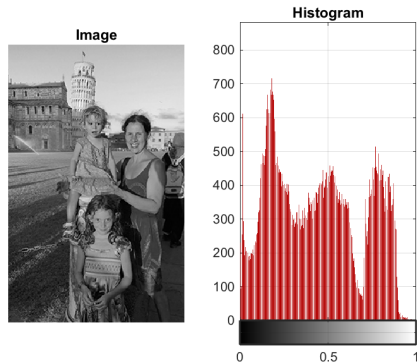
- Focus
- Exposure time correctness
- Dynamic and compression
- Noise



Histogram

Distribution

- It is independent from pixel layout
- N-Modality histograms



Entropy

Information theory: average level of information (Shannon 1948)

$$E = - \sum_i P(d_i) \log P(d_i)$$

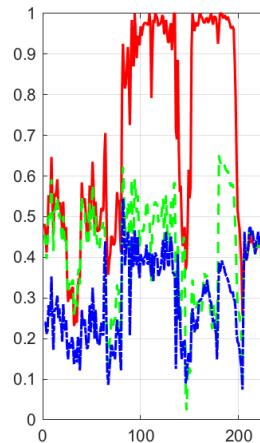
whereas P are the normalised histogram counts

e.g. it can be used for calculation of required word length in encoding

Line Intensity Distribution

Line Profiles

- In camera image processing before capturing: used for AF modules



First Order Moment

Came from center of gravity (CoG) as 1D moment and defines energy m_{ges} and CoG as d_s

$$m_{ges} = \sum_{d=d_{min}}^{d_{max}} p(d)$$
$$d_s = \frac{1}{m_{ges}} \sum_{d=d_{min}}^{d_{max}} dp(d), p, \text{ probability}$$

Higher Order Moments

Variance: 2D with standard deviation as root

$$\sigma^2 = \frac{1}{m_{ges}} \sum_{d=d_{min}}^{d_{max}} (d - d_s)^2 p(d)$$

Skewness: third order

Gray World Theory

A topic from later color section should be mentioned here:

The mean value of all image pixels is assumed to be gray.

Noise

Noise Components

- photon noise
- thermal noise
- read out noise
- amplifier noise
- quantisation

Signal to Noise Ratio

With mean pixel values μ_D and standard deviation of overall noise σ_N the Signal to Noise ratio is:

$$SNR = 20 \log_{10} \frac{\mu_D(channel)}{\sigma_N} [dB]$$

Image Distance Metric

Image Distance Metric

Distance to another image signals: Mean Squared Error (MSE), for n-channel images MSE_n is used.

$$MSE = \frac{1}{xy} \sum_{x_i=0}^{x-1} \sum_{y_j=0}^{y-1} (d_{D1}(x_i, y_j) - d_{D2}(x_i, y_j))^2$$

$$MSE_n = \frac{1}{xy} \sum_{x_i=0}^{x-1} \sum_{y_j=0}^{y-1} \sum_{n=1}^N (d_{n,D1}(x_i, y_j) - d_{n,D2}(x_i, y_j))^2$$

Image Distance Metric

Pixel to Noise Ratio (PSNR)

$$PSNR = 10 \log_{10} \left(\frac{\hat{d}}{MSE} \right) [dB]$$

Colour perception based distance metric will be discussed later.

Distance by Observer Experiments

Human Observers Comparisons under defined display conditions for geometry, time and adaptation.

In cognitive science two fundamental image comparison experiments exists:

- time sequential presentation for both images
- side by side

Some cognitive measurables: (can distinguish, cannot distinguish), (better, equal, worse), (comfortable, uncomfortable)

In simplest form two homogenous images, i.e. color patches can be compared.

Pixel Region Correlations

Constraints (neighbourhood (v,u))

$$\bar{R} = \frac{1}{N} \sum_{v=-V}^V \sum_{u=-U}^U R(u, v),$$

$$\bar{S}(x, y) = \frac{1}{N} \sum_{v=-V}^V \sum_{u=-U}^U S(x + u, y + v),$$

$$N = (2V + 1)(2U + 1)$$

Pixel Region Correlations

Correlations: Sum of Squared Differences (SSD), Sum of Absolute Differences (SAD)

$$SSD(x, y) = \sum_{v=-V}^V \sum_{u=-U}^U (R(u, v) - S(x + u, y + v))^2$$

$$SAD(x, y) = \sum_{v=-V}^V \sum_{u=-U}^U |R(u, v) - S(x + u, y + v)|$$

Normalised Correlations

Prefix: Z Zero Mean

$$ZSSD(x, y) = \sum_{v=-V}^V \sum_{u=-U}^U ((R((u, v) - \bar{R}) - (S(x + u, y + v) - \bar{S}(x, y)))^2$$

$$ZSAD(x, y) = \sum_{v=-V}^V \sum_{u=-U}^U |(R((u, v) - \bar{R}) - (S(x + u, y + v) - \bar{S}(x, y)))|$$

Normalised Correlations

Prefix: L Local Scaled

$$LSSD(x, y) = \sum_{v=-V}^V \sum_{u=-U}^U \left(R((u, v)) - \frac{\bar{R}}{\bar{S}(x, y)} S(x + u, y + v) \right)^2$$
$$LSAD(x, y) = \sum_{v=-V}^V \sum_{u=-U}^U \left| R((u, v)) - \frac{\bar{R}}{\bar{S}(x, y)} S(x + u, y + v) \right|$$

Cross Correlations

Normalized Cross Correlation (NCC)

$$NCC(x, y) = \frac{\sum_{v=-V}^V \sum_{u=-U}^U R(u, v) S(x + u, y + v)}{\sqrt{\sum_{v=-V}^V \sum_{u=-U}^U R^2(u, v) \sum_{v=-V}^V \sum_{u=-U}^U S^2(x + u, y + v)}}$$

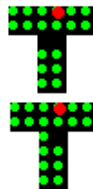
Cross Correlations

Zero Mean Normalized Cross Correlation (ZNCC)

$$ZNCC(x, y) = \frac{\sum_{v=-V}^V \sum_{u=-U}^U (R(u, v) - \bar{R})(S(x+u, y+v) - \bar{S}(x, y))^2}{\sqrt{\sum_{v=-V}^V \sum_{u=-U}^U (R^2(u, v) - \bar{R})^2 \sum_{v=-V}^V \sum_{u=-U}^U (S^2(x+u, y+v) - \bar{S}(x, y))^2}}$$

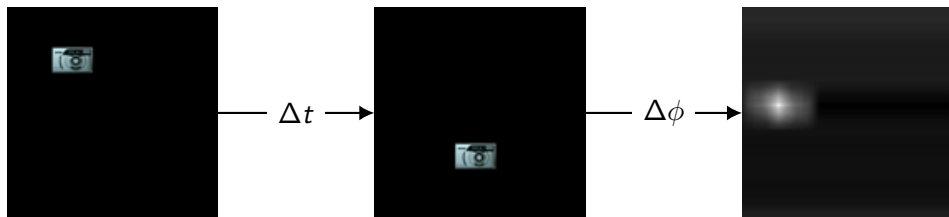
Pixel Region Correlations Sample

- Normalized correlations are robust to illumination changes
- Differentiation aspect
- Window width must be defined carefully:
SSD with small window width (right top)
vs. large window width (right bottom)



Pixel Region Correlations in Frequency Domain

Based on FOURIERs shift theorem, it can increase algorithm performance.



Co-Occurrence Matrix

How often a structure appears in some defined neighbourhood $\Delta x, \Delta y$

$$C_{\Delta x, \Delta y}(i, j) = \sum_{x=1}^n \sum_{y=1}^m \begin{cases} 1, & \Leftrightarrow I(x, y) = i \wedge I(x + \Delta x, y + \Delta y) = j \\ 0, & \text{otherwise} \end{cases}$$

Structure Similarity Index

Similarity of Luminance/Contrast/Structure Mean/Variance/Covariance (l,c,s)

$$SSIM(x, y) = l(x, y)^\alpha c(x, y)^\beta s(x, y)^\gamma$$

$$l(x, y) = \frac{2\mu_x\mu_y + c_1}{\mu_x^2\mu_y^2 + c_1}, c(x, y) = \frac{2\sigma_x\sigma_y + c_2}{\sigma_x^2\sigma_y^2 + c_2}, s(x, y) = \frac{\sigma_{xy} + c_3}{\sigma_x\sigma_y + c_3}$$

α, β, γ : weighting exponents, can be initial set to one c_i : arbitrary, but fixed constants

SSIM Sample

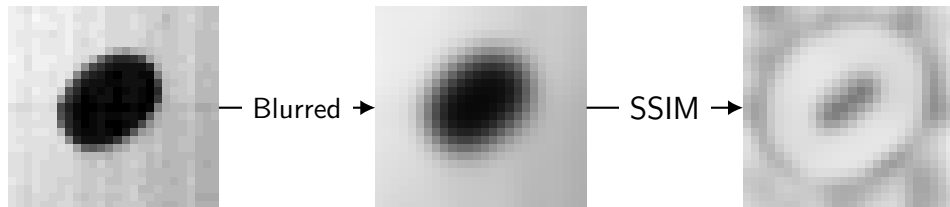
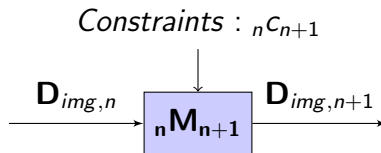


Image Processing

A Pipeline of Linear Systems



Linearity is limited by the maximum code value. In general, the image dimensions must not be the same.

Neighbourhood Operations

Convolution

The weighted sum of neighboured pixels

$$d_{conv}(x, y) = \frac{1}{w} \sum_{x'=x_l}^{x_u} \sum_{y'=y_l}^{y_u} h(x', y') d(x - x', y - y')$$

$$|x_l| = x_u = |y_l| = y_u = r$$

$$d_{conv}(x, y) = \frac{1}{w} \sum_{x'=-r}^r \sum_{y'=-r}^r h(x', y') d(x - x', y - y')$$

h is the convolution kernel

For practical reasons, it is odd, with equal horizontal and vertical dimensions.

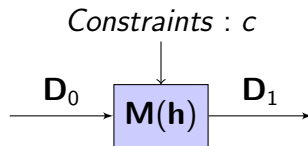
Convolution

$$\begin{pmatrix}
 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
 1 & 1 & 0 & 0 & 0 & 0 & 0
 \end{pmatrix}
 *
 \begin{pmatrix}
 1 & 0 & 1 \\
 0 & 1 & 0 \\
 1 & 0 & 1
 \end{pmatrix}
 =
 \begin{pmatrix}
 1 & 4 & 3 & 4 & 1 \\
 1 & 2 & 4 & 3 & 3 \\
 1 & 2 & 3 & 4 & 1 \\
 1 & 3 & 3 & 1 & 1 \\
 3 & 3 & 1 & 1 & 0
 \end{pmatrix}$$

D
 h
 $D * h$

System Transfer Function and Convolution Kernel

Equivalence of System Transfer Function and Convolution Kernel



- **M**: Matrix formulation of system transfer model.
- It is a function of h .
- It is a finite impulse response filter.

Convolution Theorem

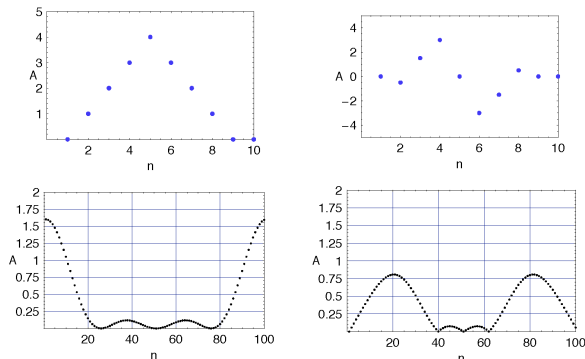
$$\begin{array}{ccc} x(t) * h(t) = y(t) \\ \downarrow \quad \downarrow \quad \downarrow \\ X(f) H(f) = Y(f) \end{array}$$

A convolution in spatial domain is equivalent to a multiplication in frequency domain.

Convolution Kernel Parameter

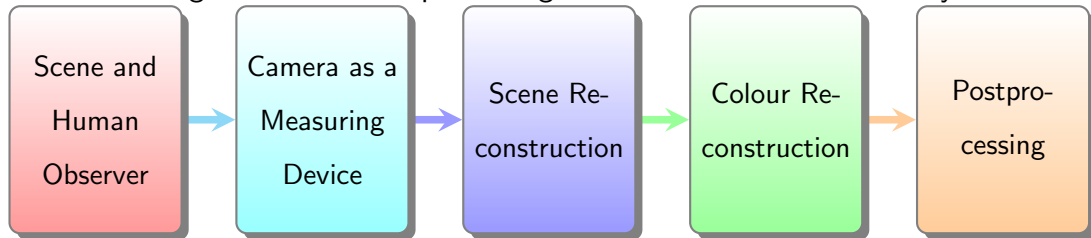
The FOURIER transformed Kernel can be specified by the amplitude response:

Low Pass (left), High Pass (right)



Digital Image Processing Operator Chain

The whole image formation and processing model can be divided in subsystems.



Non Linear Filters

So far the discussed filters are linear.

Beside that case also non linear filter exists:

$$\alpha_1 D_{1,out} + \alpha_2 D_{2,out} \neq M(\alpha_1 D_{1,in} + \alpha_2 D_{2,in})$$

2D Filter Separability

Several filter kernels can be written as a product of simpler filters, e.g. with reduced dimension.

An example for a simple average smoothing filter:

$$\frac{1}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

2D Image Reshaped in 1D Vector Notation

Another View on an Image

If we reshape an image $\mathbf{D} \in R^{m \times n}$ as $\mathbf{d} \in R^{mn \times 1}$ as an one dimensional vector, then we simply can rewrite a convolution based image processing operation in algebraic notation:

$$\mathbf{d}_1 = \tilde{M}(M(h))\mathbf{d}_0$$

which represents an equation system with a matrix-vector multiplication.

Image Formation Model

Then, if we have a captured image with a known system transfer function and $\tilde{M}$ is invertible, we are able to approximate the input of our image processing system: $\mathbf{d}_{0,est} = \tilde{M}(M(h))^{-1}\mathbf{d}_1$ by solving the minimisation problem, e.g. using the p2 norm minimum error approach $\min_{\mathbf{d}_{0,est}} \|\mathbf{d}_{0,est} - \tilde{M}\mathbf{d}_1\|_2^2$ (least square fit)

Note: remember the differentiation aspect of image distance metrics.

Model and its Inverse

The solution can be found by solving the normal equation

$$\tilde{M}^T \mathbf{d}_1 = \tilde{M}^T \tilde{M} \mathbf{d}_{0,est}$$

For unequal input and output dimensions the pseudoinverse might be calculated

$$(\tilde{M}^T \tilde{M})^{-1} \tilde{M}^T \mathbf{d}_1 = \mathbf{d}_{0,est}$$

Finding a Solution

Condition of the problem, can be well posed or ill posed, determined by the condition.

Solvers and constraints for finding a solution:

- Algebraic inverse
- Numeric solvers (LEVENBERG-MARQUARDT, Conjugate Gradient)
- Initial guess

Regularization

The Thikonov regularization is based on p2 norm with following normal equation as solution

$$(\mathbf{A}^T \mathbf{A} + \alpha \mathbf{R}^T \mathbf{R}) \mathbf{x} = \mathbf{A}^T \mathbf{y}$$

A basic approach is set $\mathbf{R}^T \mathbf{R} = \mathbf{E}$, α is a weighting coefficient. E.g. for image denoising, the assumption can be done, that sum of first derivatives also must be a minimum, then R can be set to a first derivative convolution matrix.

Processing Architecture Advantages of 1D Notation

Divide et Impera:

- Parallelization
- SIMD
- Vectorization

Finding the Model

Get the system transfer function by calibration for a model or by learning.

Model: can be physical, mathematical based. Learning: e.g. neuronal network with convolutional layers, it estimates the model function somehow.

Summary

V2 Mathematical Framework

Summary

- Image data, statistic, comparison metrics
- Image operators and system model
- 1D view on images

Take Aways

Be aware of image data domains. Choosing an appropriate operator should be provable by an image distance metric to a reference. Think about parallelization.

Day 4

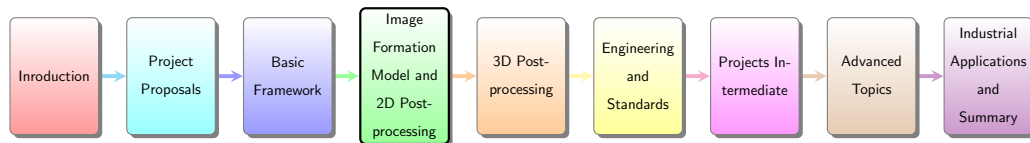
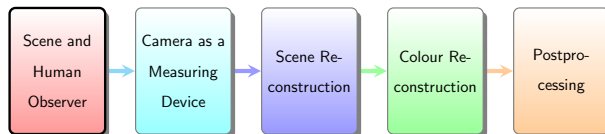


Image Formation and Processing

V3 Physical Properties of Scene and Human Observer



Content I

4 V3 Physical Properties of Scene and Human Observer

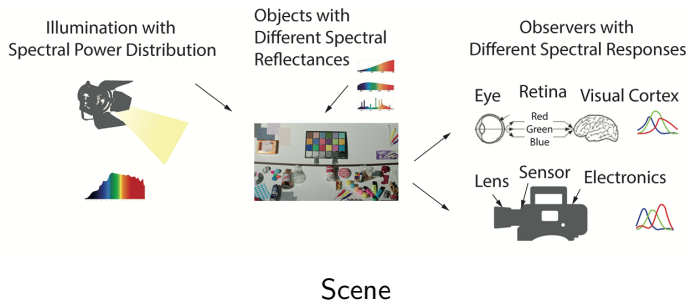
- Scene
- Illuminations
- Objects
- Photoreceptor
- Human Observer

Scene Composition and Human Observer

- Illuminations, natural and artificial light
- Objects, how they reflect the light
- Human Eye Image Formation, how the object is seen

Scene

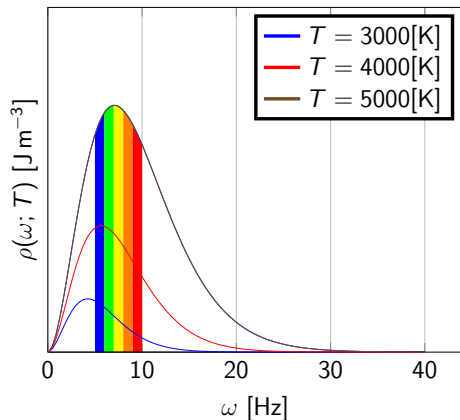
Scene and its Components



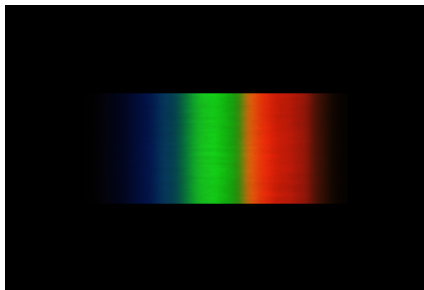
Illuminations

Spectral Power Distribution (SPD)

- Black-Body-Radiator
- Model by PLANCKs law based on thermodynamic statistics
- Depends on temperature
- With $\omega \sim \lambda^{-1}$



Spectral Power Distribution

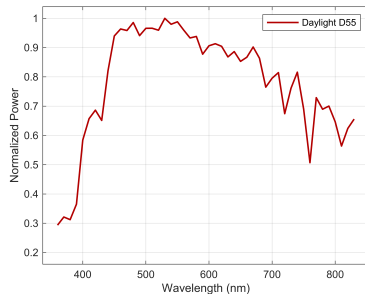


Spectrum between 400 and 800
nm

- In general it is described as Power $\Phi(\lambda)$ or equivalent per wavelength.
- Spectral radiance $L(\lambda)$ is one important equivalent.
- Measured data samples Φ with $\Delta\lambda = 1..30$ nm
- Sampling problem for lights with narrow bands

Daylight (Sunlight)

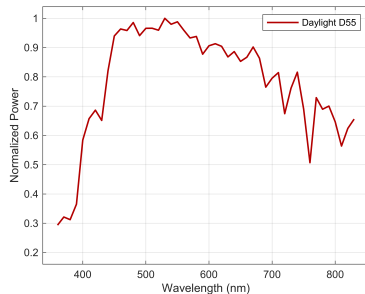
- Sunlight influenced by atmospheres
Rayleigh scattering $\sim \frac{1}{\lambda^4}$ and
absorption.
- Late morning in South England
- Can be approximated by a broken
rational polynomial.



D55 Daylight

Daylight (Sunlight)

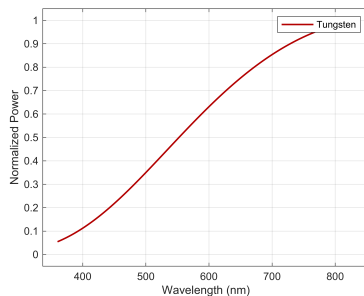
- Standard illuminant D series, e.g. D55
- Note: role of atmosphere
scattering/absorption in remote
sensing/underwater



D55 Daylight

Tungsten (Light Bulb Artificial Light)

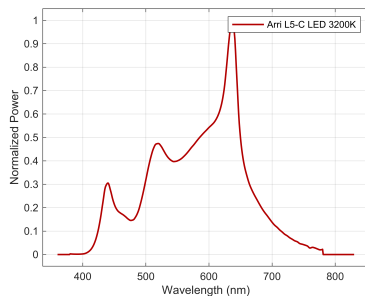
- Commonly used light source in last century
- Almost linear except boundary regions
- Standard illuminant A
- Follow PLANCK's law, but has glass bulb absorption



Tungsten SPD for $T=3200K$

LED as State of the Art Light

- pn transition radiation, generalized PLANCKs law
- SPD-Approximation by Gauss distribution
- Superposition of different (RGB) LEDs

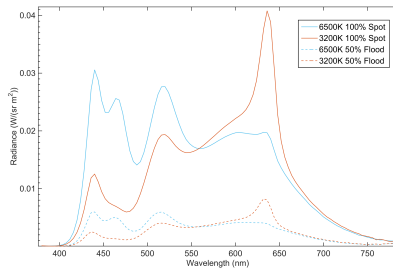


LED Tungsten Simulator

LED Sample

Technology
Arts Sciences
TH Köln

- Simulating by weighted electric current
- Temperature drift



Different LED Lights

Standard Illuminants and Simulators I

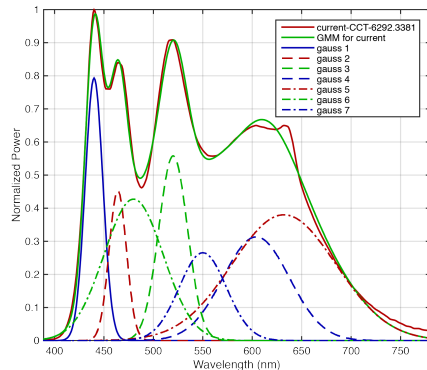
- In photography and movie industry a set of standard illuminant exist.
- They represent natural and artificial light sources with SPDs.
- Manufacturerers offers so called standard illuminant simulators, which approximates the SPDs of the references.
- The SPD is much important even before images are captured, because humans visually evaluate the quality of illumination on set.

Standard Illuminants and Simulators II

- SPDs can be approximated by parameters set, with much less members the the amount of spectral samples.

LED Approximation

- Superposition of different (RGB) LEDs but also often..
- ..secondary white emission
- Best fit contradicts to physical reality



GAUSS approximations

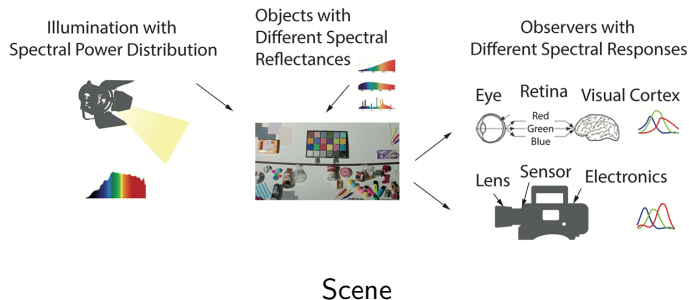
Illumination Setups

- diffuse/direct
- direct/indirect
- reflectance/transmittance
- bright/dark field lighting
- Phase Contrast

Objects

Scene Overview

The self-luminous light source emits light, which hits an object surface.



Object Surface

Object surface can reflect, transmit and absorb light.

- Reflectance $\rho(\lambda)$: commonly used in photo/movie
- Transmittance $\tau(\lambda)$: important in microscopy/reproduction
- Absorption $\alpha(\lambda)$: lenses, color filters

Note: FRESNEL reflection model can be used for absorption free dielektrika

Spectral Reflectance

The illumination radiance SPD is the irradiance to an object. It is reflected by an illuminated object. Spectral reflectance $\rho(\lambda) = \frac{\Phi_1(\lambda)}{\Phi_0(\lambda)}$

Measured data samples with $\Delta\lambda = 1..30$ nm

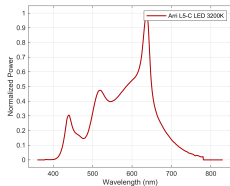
Now the surface acts as a not self-luminous radiation source.

$$\Phi_1(\lambda) = \text{diag}(\rho(\lambda))\Phi_0(\lambda)$$

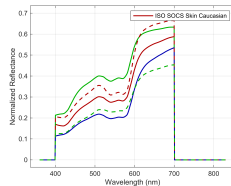
For transmittance and absorption ρ can be replaced with τ , α respectively.

Radiation from Illuminated Object

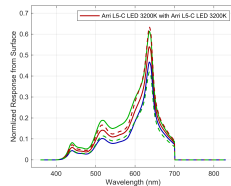
$$\Phi_1(\lambda) = \text{diag}(\rho(\lambda))\Phi_0(\lambda)$$



— Φ →



— $\rho\Phi$ →



Numerical Problems

- Different sampling of Φ and ρ , interpolation artefacts
- Different boundaries, continuity assumptions, extrapolation artefacts

Surface Characteristics

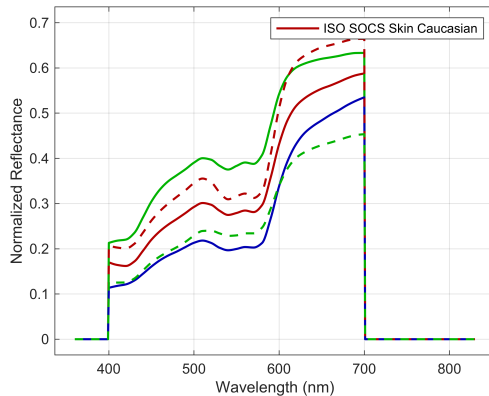
- LAMBERT characteristic: complete diffuse reflection
- Gloss/Glare as an additional property

Mix of diffuse and glossy reflection (w_i , weighting)

$$\Phi_1 = w_d \rho \Phi_0 + w_h \Phi_0$$

Reflectance Sets

- Huge reflectance diversity/manifold in nature, sample sets exist:
- White reference
- ISO SOCS (right excerpt)
- Charts...



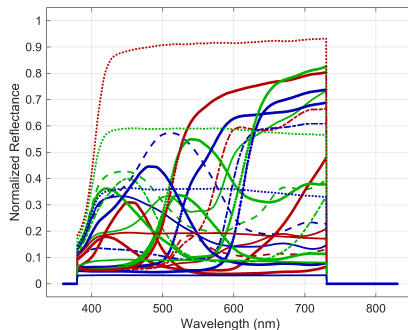
Caucasian Skin Tones

Reflectance Charts

Random samples for skin tones, and saturated base colors.



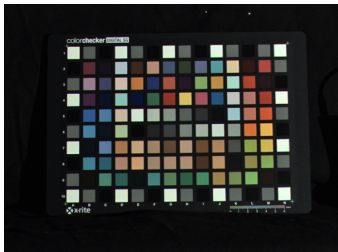
Color Checker



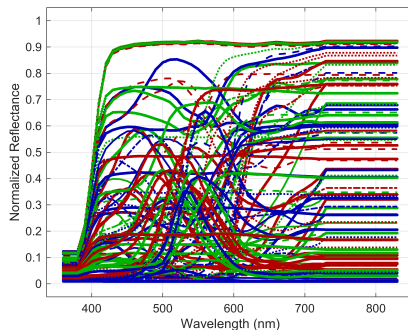
Spectral Reflectances

Reflectance Charts

Superset of ColorChecker, contains also glossy patches.



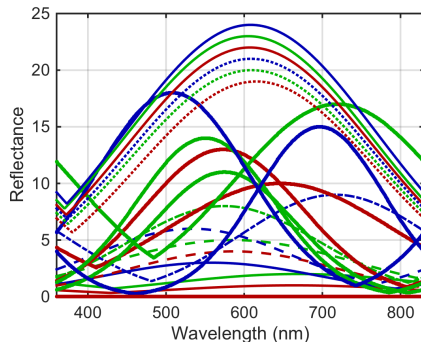
Color Checker Digital SG



Spectral Reflectances

Synthetic Spectra Set

- Mathematical models for reflectances
- e.g. LOGVINENKO, $\kappa\sigma\mu$ model
- Notice the discontinuity
- Can be used for color management (following lectures)

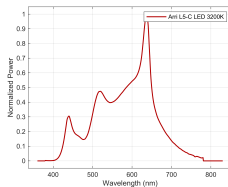


Set of Synthetic Spectra

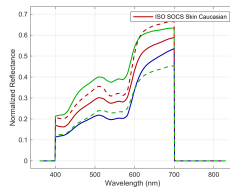
Photoreceptor

Recap: Radiation from Illuminated Object

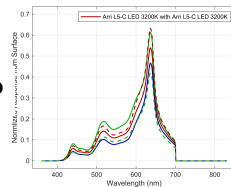
$$\Phi_1(\lambda) = \text{diag}(\rho(\lambda))\Phi_0(\lambda)$$



— Φ →

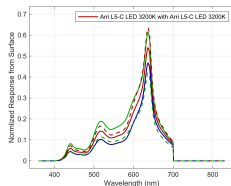


$\text{diag}(\rho)\Phi$

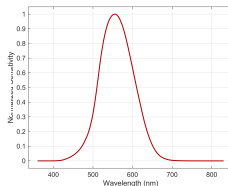


Photoreceptor Signal Formation

- Spectral sensitivity/response $\mathbf{S}(\lambda)$
- Registered signal: $d = \text{diag}(\Phi(\lambda))\rho(\lambda)^T \mathbf{S}(\lambda)$
- Sensitivity cuts off irradiance power



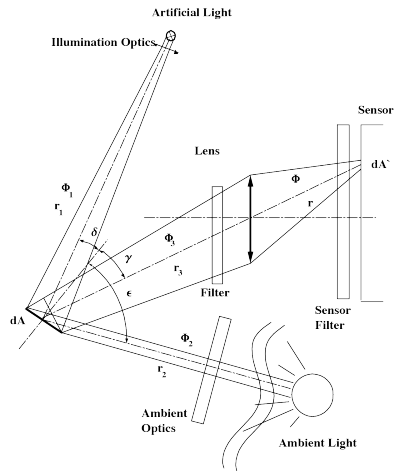
$$\text{diag}(\rho)\Phi$$



$$\longrightarrow \langle \text{diag}(\Phi(\lambda))\rho(\lambda), \mathbf{S}(\lambda) \rangle \longrightarrow d$$

Complex Scene Radiometry

- Complex scene radiometry
- Superposition of illuminations
- A sensor mostly has a lens in front to collect light
- Side by Side sensors are able to collect light of side by side surface elements.



Light Observers in Nature I

- Lens/Photoreceptor/Neural Network Combination
- Lenses can be formed by arrays
- Photoreceptors with different spectral sensitivities, often optimized for daylight
- Reduction of spectral power distribution
- Slightly compensated b using more the one photoreceptor type, e.g. octopus, insects ...

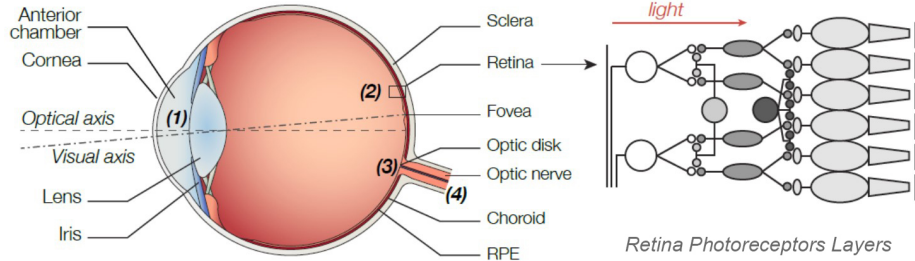
Light Observers in Nature II

- most applications are aimed at human observer...
- Reduction of 3D world to 2D, slightly compensated by using more than one eye
- Not registered physical properties of light: amplitude, polarization, phase

Human Observer

From Eye to Retina

Retina is the photoreceptor layer.



©Wikimedia

Optical Elements

In front of retina:

- Pupil/Iris: responsible for adaptation
- Cornea/Lens: responsible for accommodation
- Vitreous humor

Optical system have errors, which human vision corrects at higher level, more on lens errors in next lecture.

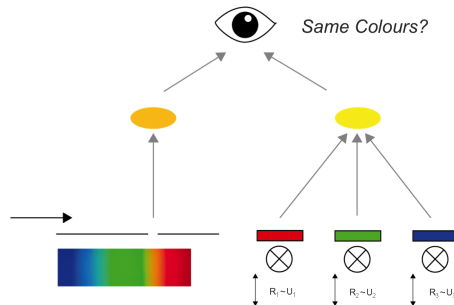
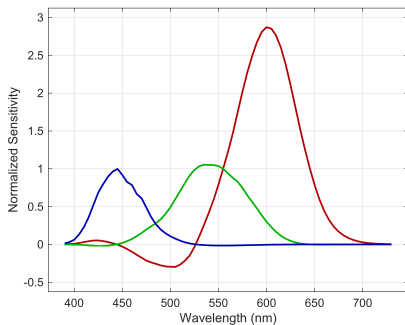
Human Eye Photoreceptor Types

Side by side receptor cells in retina:

- Cone cells, 3 types "Blue" (S), "Green" (M), and "Red" (L), photopic and colour vision at day
- Rod cells, scotopic gray vision at night
- Dawn mixed vision
- Spectral sensitivities in range [360,830] nm (VIS)

Observer Response

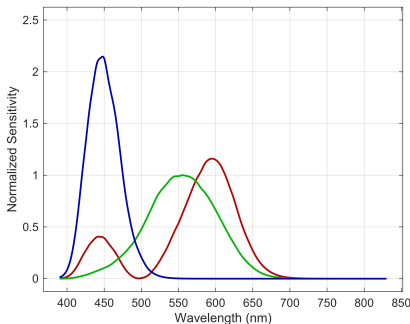
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From comparison experiment: $\bar{r}, \bar{g}, \bar{b}$

But can be negative!

Standard Observer Response



- CIE standard observer (1936)
- Now the linear transformed response is always positive: $\bar{x}$, $\bar{y}$, $\bar{z}$
- Due to genetic variations observer variability exists.

Colour as Tristimulus

Using $(\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda))$ following defines a color tristimulus:

$$\vec{C}_{Obs} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = k \begin{pmatrix} \int_{\lambda=360nm}^{830nm} \phi(\lambda) \rho(\lambda) \bar{x}(\lambda) d\lambda \\ \int_{\lambda=360nm}^{830nm} \phi(\lambda) \rho(\lambda) \bar{y}(\lambda) d\lambda \\ \int_{\lambda=360nm}^{830nm} \phi(\lambda) \rho(\lambda) \bar{z}(\lambda) d\lambda \end{pmatrix} \approx k \begin{pmatrix} \langle \text{diag}(\Phi(\lambda)) \rho(\lambda), \bar{\mathbf{x}}(\lambda) \rangle \\ \langle \text{diag}(\Phi(\lambda)) \rho(\lambda), \bar{\mathbf{y}}(\lambda) \rangle \\ \langle \text{diag}(\Phi(\lambda)) \rho(\lambda), \bar{\mathbf{z}}(\lambda) \rangle \end{pmatrix} \quad (1)$$

k Luminous efficacy

Retina Geometry

- Spherical form
- Highest resolution at yellow spot: **vernier acuity** with approx. 1'
- Lateral decreasing spatial resolution
- In computer vision it must be ensured that every point in presented image offers the highest resolution according to **vernier acuity**, because humans have a saccadic view at points of interest

From Retina to Visual Cortex

The registered photoreceptors image is transmitted to visual cortex. Then the set of tristimuli leads to:

- Scene dynamic perception
- Flicker perception
- Colour perception

Dynamic Perception

- Scene dynamic in nature very high
- Static cone dynamic very limited, additional pupil adaptation
- Main important thing: logarithmic signal perception....

WEBER-FECHNER Law

- Important law in psychophysics
- Relation between change in physical stimulus and perceived change
- Ordinary differential equation and its solution:

$$dp = k \frac{dS}{S} \Rightarrow p = k \ln S + C$$
$$p = k \ln \frac{S}{S_0}, C = -k \ln S_0$$

Flicker Perception

- Flicker fusion threshold
- Depends on radiance
- e.g. 25-50 Hz for adults, or higher for babies
- Important for image sequence processing and presentation

Colour as Perception

- *For Photography and Movie Production:*
 - *Does your Camera image looks like my camera image?*
- *For Classification in Machine Vision:*
 - *Are your Camera RGB pixel values the same as for my camera/illumination?*

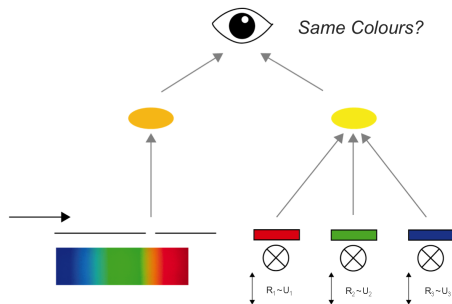
The answer is in general *No*, therefore we need some methods to map colours from different camera systems into a reference, defined by human observer.

- Colour, the perception:
 - DIN 5033-1 [1] „durch das Auge vermittelter Sinneseindruck, durch den sich zwei aneinandergrenzende, strukturlose Teile des Gesichtsfeldes bei einäugiger Betrachtung mit unbewegtem Auge allein unterscheiden lassen“

Colour as Tristimulus

- The perception is caused by a tristimulus $\mathbf{C}$, since the human eye is a trichromatic system (Young 1801): a colour can be represented by a mix of three basic colours
- Colour mixing experiments (Maxwell 1855-1860): a colour mixed by three basic colours can be linear transformed into another three basic colours mixture
- Experiment for deriving the RGB sensitivities of human eye: the colour matching functions (Wright 1929), $(\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda))$ remixing a monochromatic light by three coloured lights with variable intensities

$$\vec{C} = \begin{pmatrix} R_2 \\ G_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} R_1 \\ G_1 \\ B_1 \end{pmatrix}$$

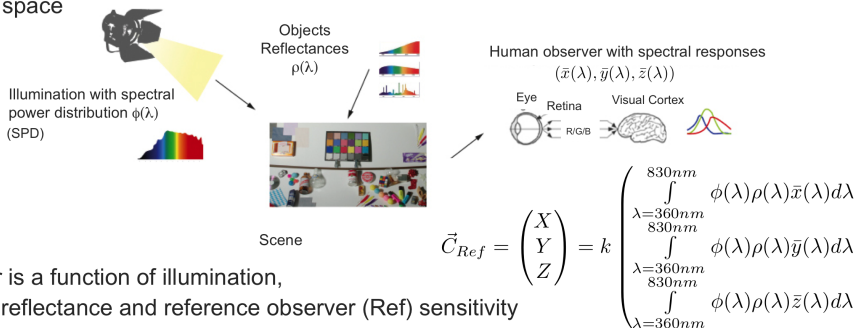


Standardized Colourimetric Domain

- Human Standard Observer (2° and 10°) defined 1931 by CIE (Commission Internationale d l'Eclairage) colour matching functions, as a constrained linear combination of colour matching functions

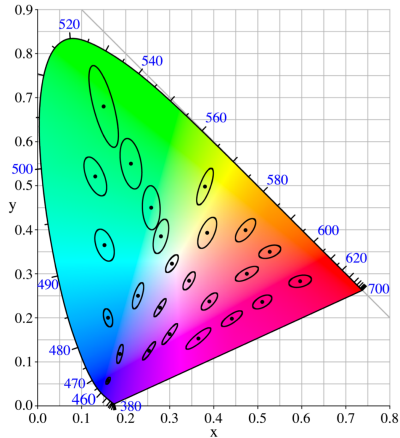
$$(\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)) = f((\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)))$$

- Standardized quantitative colour description for object colours as tristimulus in CIE XYZ colour space



- Colour is a function of illumination, object reflectance and reference observer (Ref) sensitivity

Colour Differences



- Chromaticities (left chromaticity diagram)

$$x = \frac{X}{X + Y + Z}, y = \frac{Y}{X + Y + Z}, z = \frac{Z}{X + Y + Z}$$

- Mac Adams 1942, experiment for just noticeable difference (JND), Mac Adams Ellipses (left figure)
- CIE XYZ is not perceptual equidistant, same perceptual colour differences have different Euclidian distances in CIE XYZ domain

https://de.wikipedia.org/wiki/MacAdam-Ellipse#/media/File:CIExy1931_MacAdam.png

Perceptual Domain and Colour Difference

- Colour Appearance Model (CAM), perceptually uniform colour space

- Quantitative description in perceptual domain
split into lightness and chrominance:

CIE 1976 $L^*a^*b^*$ (CIE-Lab)

- $(X_n Y_n Z_n)^T$ as „White Point“
- Mac Adams Ellipsis now nearly circles of same diameter

$$L^* = 116f_{Lab}(Y/Y_n) - 16$$

$$a^* = 500 (f_{Lab}(X/X_n) - f_{Lab}(Y/Y_n))$$

$$b^* = 200 (f_{Lab}(Y/Y_n) - f_{Lab}(Z/Z_n)).$$

$$f_{Lab}(x) = \begin{cases} x^{\frac{1}{3}}, & \text{if } x > (\frac{6}{29})^3 \\ x^{\frac{841}{108}} + \frac{4}{29}, & \text{if } x \leq (\frac{6}{29})^3 \end{cases}$$

- Distance metric: Delta E 1976 colour difference $\Delta E = \sqrt{(L_1^* - L_2^*)^2 + (a_1^* - a_2^*)^2 + (b_1^* - b_2^*)^2}$
- Further optimized in CIE Delta E 2000 formulae

Colour Constancy

- Colour constancy: invariance of colour appearance despite changes in illumination, “white point preserving”
 - Short time (visual cortex) “discounting illuminant” cognitive mechanism
 - Long time (retina) “chromatic adaptation”: photoreceptors are able to adjust their sensitivity in response to ambient illumination
 - *Open Question:*
 - *It is not clear what determines an observer’s white point.*
- Computational colour constancy:
 - illumination estimation
 - illumination independent colour descriptor

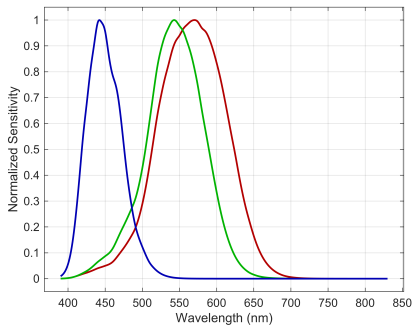
Colour Constancy

- linear model as in von Kries cone domain scaling, assumption that cone responses are linear combination of cone matching fundamentals
- Transformation of tristimulus illuminated under two different lighting conditions with
M:= cone matrix, A:= cone adaptation matrix, M_{CAT}:= chromatic adaptation transform

$$\begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = M^{-1} A M \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = M_{CAT} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} \quad A = \begin{pmatrix} \frac{L_{W2}}{L_{W1}} & 0 & 0 \\ 0 & \frac{M_{W2}}{M_{W1}} & 0 \\ 0 & 0 & \frac{S_{W2}}{S_{W1}} \end{pmatrix} \quad \begin{pmatrix} L_{Wi} \\ M_{Wi} \\ S_{Wi} \end{pmatrix} = M \begin{pmatrix} X_{Wi} \\ Y_{Wi} \\ Z_{Wi} \end{pmatrix}, i = 1, 2$$

- M_{CAT} is a result of linear optimization (Lam/Bradford University), M is derived via Eigenvalue decomposition
- Non linear models

LMS Cone Space



- White adaptation applied cone space

Metamerism

- Metamerism: spectrally different stimuli have same colour appearance, i.e. identical cone response, fundamental for colour reproduction by different primaries
- If a pair of metameric colours under one set of conditions differs:
 - Viewed under different light: illuminant metamerism
 - Viewed by different observer: observer metamerism
 - Viewed by different field size: field size metamerism

Other Colour Perception Phenomena

- Adaptation: luminance and chromatic adaptation
- Simultaneous contrast: an area's colour is affected by the immediate surrounding colour
- Crispening: magnitude of perceived colour difference of two stimuli on one background increases if background is similar to stimuli
- Spreading: colours appear to blend with increasing spatial frequency

Other Colour Perception Phenomena

Colour:

- HELMHOLTZ-KOHLRAUSCH: perceived brightness increases with saturation and is hue dependent
- HUNT: perceived colourfulness increases as the overall luminance of image increases

Contrast:

- BARTLESON-BRENEMANN: perceived image contrast increases as the luminance of surrounding field increases
- STEVENS: perceived contrast increases as the overall luminance of image increases

Colour affected by Monochromatic illumination:

- BEZOLD-BRÜCKE: perceived hue of monochromatic lights changes with intensity (Hill, CRT to LED to Laser?)
- ABNEY: mixtures of monochromatic with white light do not maintain constant hue
- HELSON-JUDD: gray objects on gray background illuminated with monochromatic light seems to have hue and chroma; if lighter than background having same hue, if darker than background having complimentary hue as light source

Colour Models Be Aware of Higher Level Effects

- Related Colours
 - Real world and images
- Unrelated Colours
 - Isolated in experiment, but for reproducible quantitative colour description we must try to reduce/eliminate effects by strength viewing constraints
- Colour Appearance Models (CAM)
 - CIECAM02, including two additional surrounding areas of different lightness
- Image appearance models (IAM)
 - iCAM04, be aware of local spatial contrast

Summary

V3 Physics of Scene and Human Observer

Summary

- Every light source has its SPD
- Every object reflects light dependend to wavelength
- Human beings have 3 colour related photoreceptors, each integrates the object flux.

Take Aways

Wherever you see a colour image, think about illumination and reflectance spectra and colour perception.

Day 5

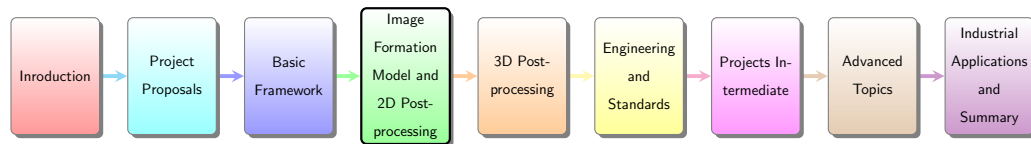
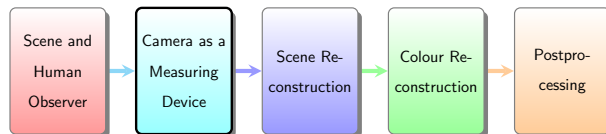


Image Formation and Processing

V4 The Camera System as a Measure Device



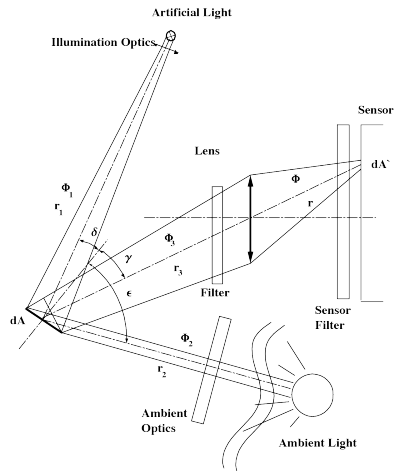
Content I

- 5 V4 The Camera System as a Measure Device
 - Geometric Camera Model
 - Lenses, Geometric Image Formation and Errors
 - Sensor, Image Signal Formation and Errors
 - Other Errors

- Learn the principal components of a camera device
- Study an image formation model of a camera
- Get in touch with errors by lenses and sensors

Camera as a Measurement Device

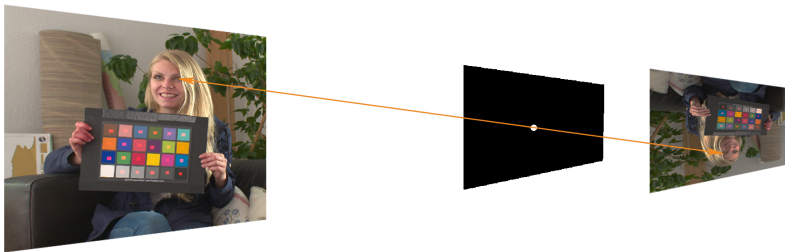
- Integrates the irradiance over exposure time and pixels area
- Continuous radiance distribution of scene is sampled by the sensors rectilinear grid of pixels
- Depending on underlying model, a pixel counts photons or integrates electromagnetic field amplitudes



Geometric Camera Model

Pin Hole Camera - Camera Obscura

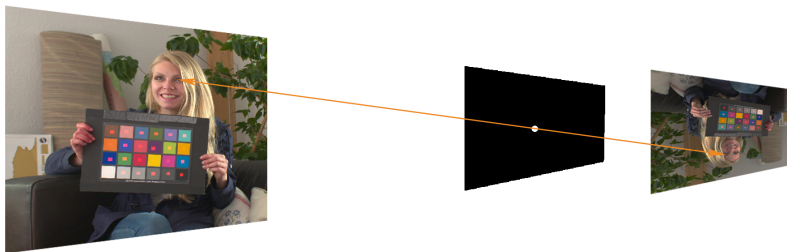
Simplest camera system and simplified camera model (ARISTOTELES 400 BC)



Pinhole Camera

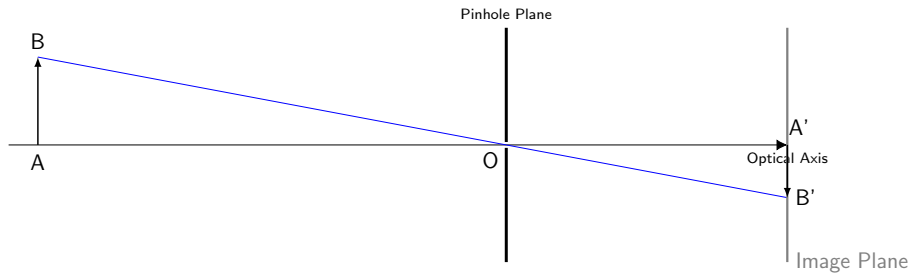
Pin Hole Camera - Camera Obscura

Applying a hole with diameter comparable to wavelength, image is formed by constructive interference of object wavefronts (HUYGENS 1678/FRESNEL 1816), used by painters even before photosensitive materials were available



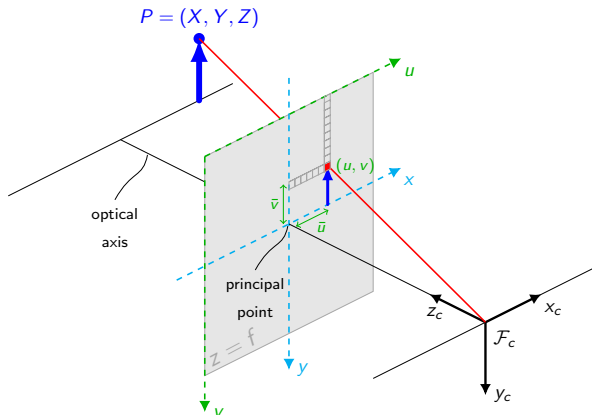
Pin Hole Camera - Geometry

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3D Pin Hole Camera Geometry

Image plane is swapped:



Camera Equation

The projection is followed by 180 rotation and uses one coordinate system for camera and object. Based on proportionality theorem the coordinate relation is:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{-f}{Z} \begin{pmatrix} X \\ Y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix} - \begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix}$$

f is called focal length as distance from pin hole to center of image plane, with relation to image plane coordinates (u,v) and center at $(\tilde{u}, \tilde{v})$

Camera Equation

Generalized camera equation in homogeneous coordinates:

$$\mathbf{r}_{Cam} = \begin{pmatrix} x_{Cam} \\ y_{Cam} \\ 1 \end{pmatrix} \sim \mathbf{C} \begin{pmatrix} x_{World} \\ y_{World} \\ z_{World} \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix}, \mathbf{C}: \text{Camera Matrix}$$

It represents the relation between scene/world 3D and camera/image plane 2D coordinates. It can be used for coordinate measurements.

Geometrical Resolution of Pinhole

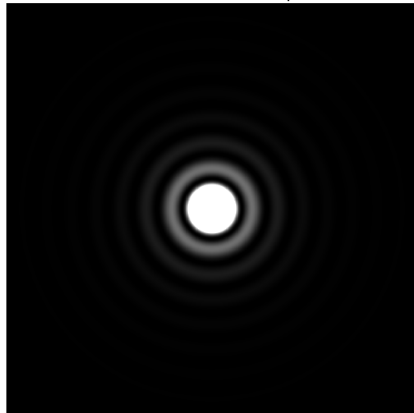
...but there is no real infinitesimal image point location, it is blurred by:

- Interference pattern (diffraction limited)
- Bessel function of first order for circular pin holes
- It depends on wavelength
- In general it is formed by white light interference

Geometrical Resolution of Pinhole

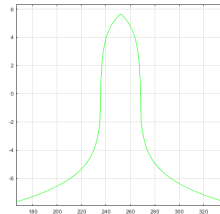
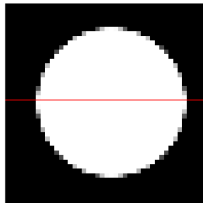
- Point Spread Function (PSF) as system transfer $\mathbf{d} = \mathbf{M}_{PSF} \Phi$
- Diffraction degenerated input pulse: AIRY disc
- RAYLEIGH criterion
$$\sin \theta \approx \theta \approx 2r_{Airy}/f \approx 1.22\lambda/D$$
- e.g. $5 - 10\mu m$ for light with a f-stop f/D 1/8..11

Fraunhofer Diffraction of circular aperture



Geometrical Resolution of Pinhole

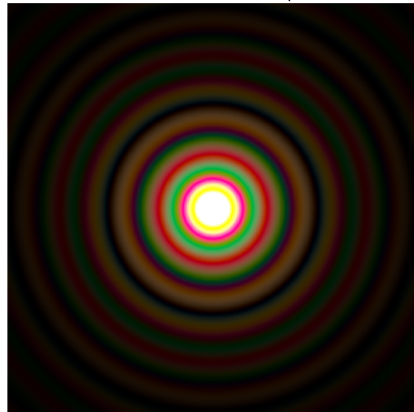
- Amplitude spectrum in Fourier domain
- It discriminates higher frequencies



Geometrical Resolution of Pinhole

- But normally the object flux is a broadband SPD or "mixed of colours"
- It results in an overlay of coloured AIRY disks

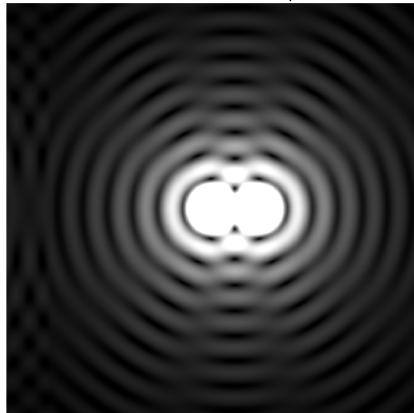
Fraunhofer Diffraction of circular aperture



Geometrical Resolution of Pinhole

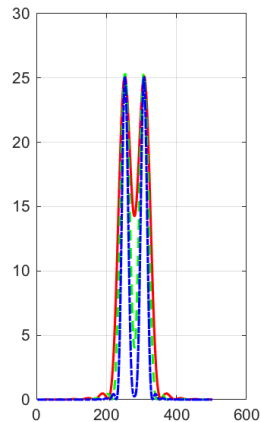
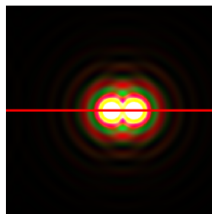
- RAYLEIGH criterion for minimum distance resolution
- Full Width Half Maximum (FWHM) distance
- It limits the measurement of coordinate differences

Fraunhofer Diffraction of Circular Aperture



Geometrical Resolution of Pinhole

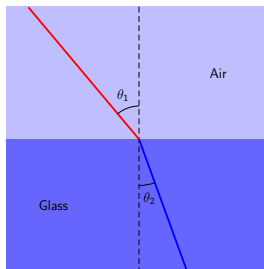
- Rayleigh criterion
for resolution with
broad band
illumination
spectrum



Lenses, Geometric Image Formation and Errors

From Pinhole to Lens

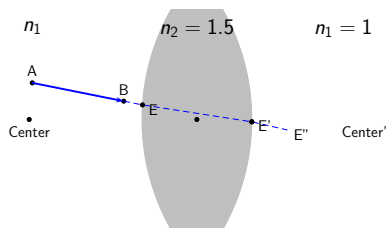
SNELLIUS 1621: light ray propagation law $n_1 \sin \theta_1 = n_2 \sin \theta_2$



n : refractive index $n = 1$ vacuum, $n \approx 1$ air, $n = 1.33$ water

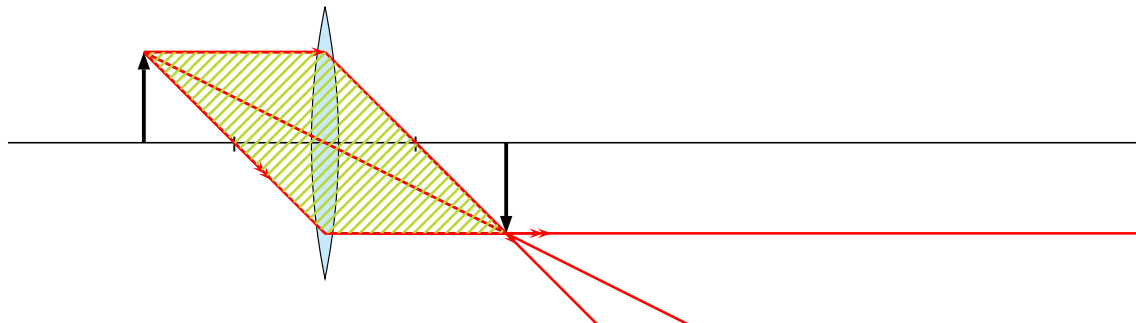
Using SNELLIUS Law for Lenses

Can be used to model lenses as image formation components, $n_{Glass} \approx 1.3..1.9$



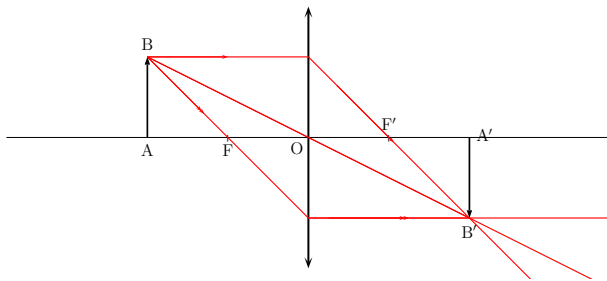
Lens instead of Pinhole

Collects more flux power, and enables us to have different magnification/form factors



Lens Image Formation Model

Mapping equation $\frac{1}{f} = \frac{1}{b} + \frac{1}{g}$ with focal length $f = \overline{OF}$, image distance $b = \overline{OA'}$, object distance $g = \overline{AO}$, image scale $\beta = \frac{\overline{OA'}}{\overline{AO}} = \frac{\overline{B'A'}}{\overline{BA}}$



Lens Model

Lensmakers equation, whereas radius is the important production factor

$$\frac{1}{b} + \frac{1}{g} = \frac{1}{f} = (n - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{(n - 1)d}{nR_1R_2} \right)$$

$n-1$ stands for lenses in air, in general it is $n_{lens} - n_{medium}$

Thin Lens Approximation

If thickness d is negligible:

$$\frac{1}{b} + \frac{1}{g} = \frac{1}{f} \approx (n - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

Matrix Optic

Ray tracing in par-axial area $n_i \sin \theta_i \approx n_i \theta_i$ r : height above optical axis α : angle to optical axis

$$\begin{pmatrix} r_2 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_1 \\ \alpha_1 \end{pmatrix}$$
$$\begin{pmatrix} r_{out} \\ \alpha_{out} \end{pmatrix} = \prod_{i=n}^1 T_i \begin{pmatrix} r_{in} \\ \alpha_{in} \end{pmatrix}, T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Lens Matrices

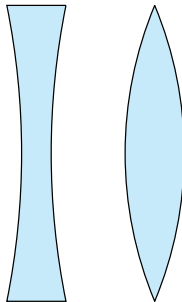
Thin/Thick Lens:

$$T = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 - \frac{d}{R_1} \frac{n_L - 1}{n_L} & d \frac{1}{n_L} \\ (n_L - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{(n_L - 1)d}{n_L R_1 R_2} \right) & 1 + \frac{d(n_L - 1)}{R_2 n_L} \end{pmatrix}$$

Lens Types

Surfaces can have concave/convex/plan surfaces.



Biconcave(left) and Biconvex (right) samples above

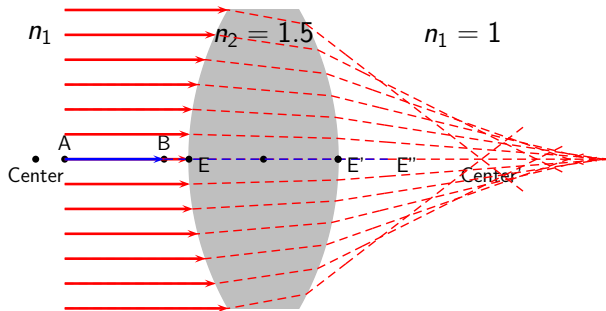
Lens Errors

Now we can use lenses and having short exposure times, but we pay this with lens caused image formation errors, i.e. aberrations:

- Spherical aberration (spherical lens less production requirements)
- Chromatic aberration (dispersion)
- Coma

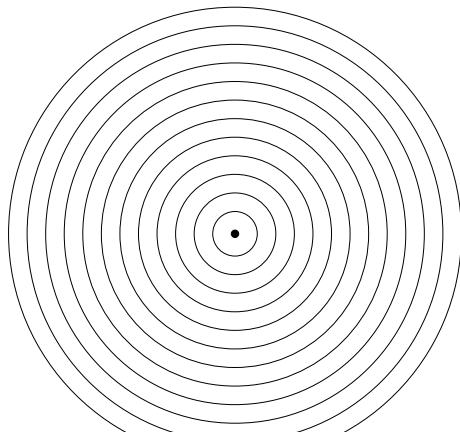
Spherical Aberration

The higher the incidence angle, the shorter the focal length



Spherical Aberration

Image Plane Intensity Distribution, Lines of Equidensity



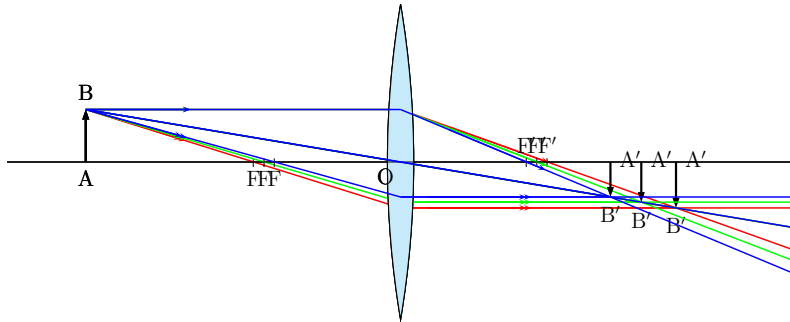
Dispersion

Due to dispersion the focal length depends on wavelength

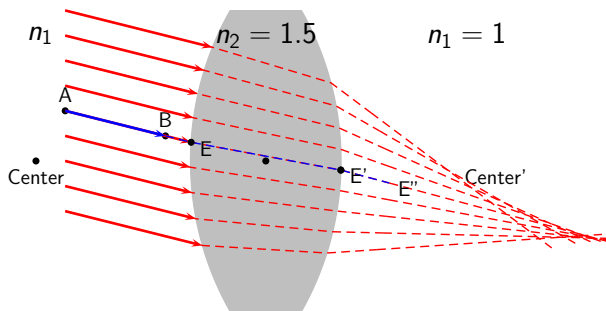
$$\frac{1}{f(n_\lambda)} = \frac{1}{b} + \frac{1}{g}$$

Chromatic Aberration

And dispersion leads to chromatic aberration. Note the different focal lengths and image location!

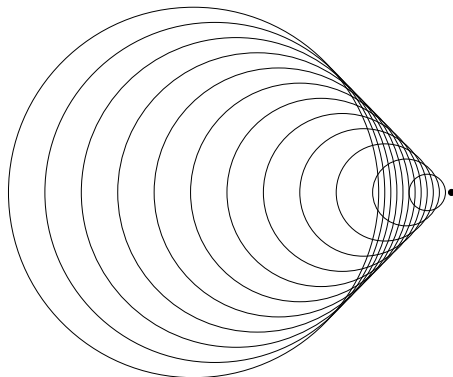


Coma



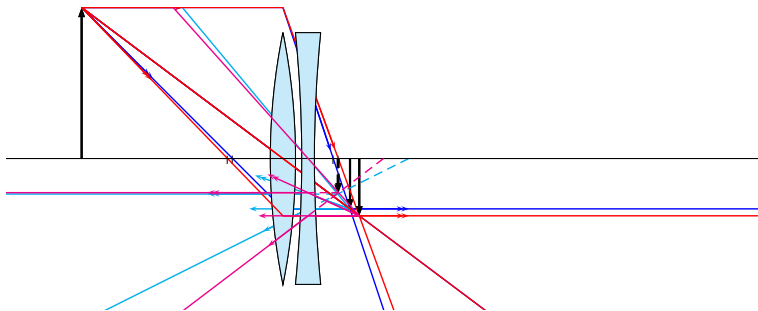
Coma

Image Plane Intensity Distribution, Lines of Equidensity



Error Reduction by Lens Design

Combined convex and concave lenses, also it has a smaller form factor, example of simple achromat:

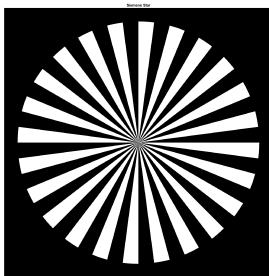


Overall Lens Errors

- Diffraction errors much less than geometric optic errors
- Lens system errors in sum too complex
- We use charts to measure/model errors...

Chart for Lens Transfer Evaluation

SIEMENS star



— Camera System —>

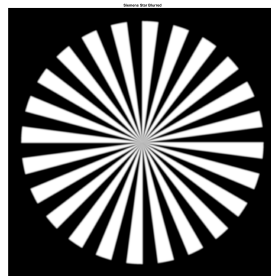


Image Information Transfer Evaluation

- Contrast $C = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \sim \frac{d_{max} - d_{min}}{d_{max} + d_{min}}$
- Modulation Transfer Function MTF
- MTF depends on light wavelength, field position, spatial orientation, focal length and aperture value.
- Contrast/Modulation by spatial frequency
- For lens only, sensor only or combined for whole camera system

Chart for Information Transfer Evaluation



- Charts of continuously varying frequency
- MTF is the magnitude of the Fourier transform of the point or line spread function, the response of an imaging system to an infinitesimal point or line of light.
- Depends on SPD of illumination!

Image Information Transfer Evaluation

Definition of MTF based on sine patterns: MTF is the contrast at a given spatial frequency ν relative to contrast at low frequencies:

- d_B : minimum luminance (or pixel value) for black areas, at low spatial frequencies; frequency should be low enough, that contrast doesn't change if it is reduced.
- d_W : maximum luminance for white areas, at low spatial frequencies.
- d_{min} : minimum luminance for a pattern near spatial frequency ν ("valley").
- d_{max} : maximum luminance for a pattern near spatial frequency ν ("peak").

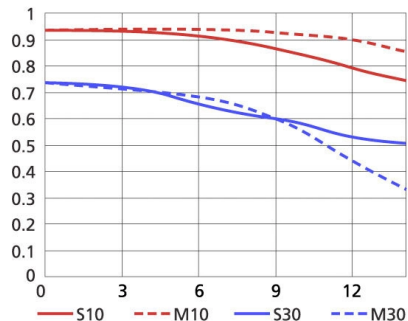
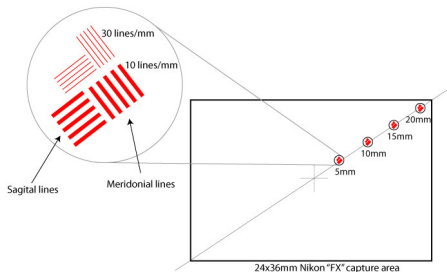
Image Information Transfer Evaluation

- $C(0) = (d_W - d_B) / (d_W + d_B)$ is the low frequency (black-white) contrast.
- $C(f) = (d_{max} - d_{min}) / (d_{max} + d_{min})$ is the contrast at spatial frequency ν .
Normalizing: dividing by $d_{max} + d_{min}$ ($d_W + d_B$ at low spatial frequencies),
minimizes errors due to gamma-related non linearities in acquiring the
pattern.
- $MTF(\nu) = 100 \frac{C(f)}{C(0)}$

MTF Example

Technology
Arts Sciences
TH Köln

AF-S DX NIKKOR 35mm f/1.8G, ©Nikon Inc.



Spatial Frequencies	S: Sagittal	M: Meridional
10 lines/mm		
30 lines/mm		

Image Information Transfer Evaluation

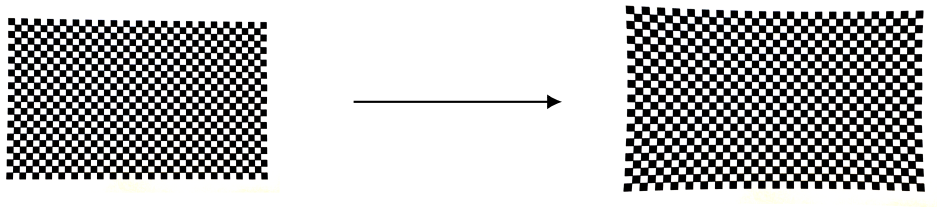
The MTF can be used to describe any kind of imaging component:

Hardware/Software

- A lens (system), as discussed
- A sensor or ...
- ... the whole camera system
- And even every image processing system, like an algorithm
- MTF application will be discussed for original image reconstruction later on

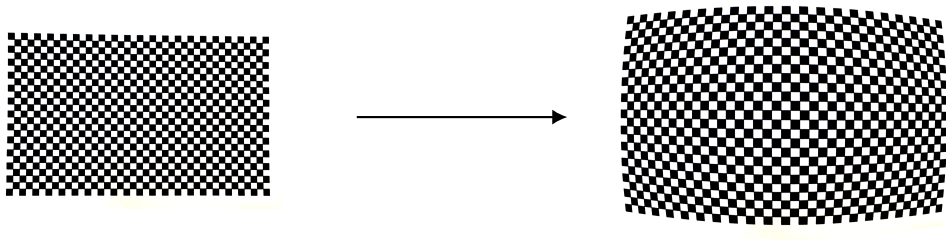
Geometric Degradation: Distortion

Another degradation in image formation is distortion. a) Pincushion



Distortion

b) Barrel



Distortion

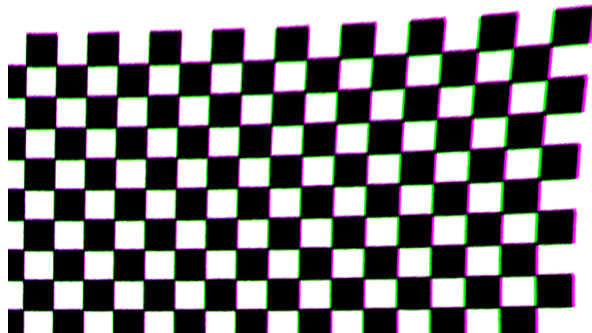
It is a lens pupil related error, a commonly used rotation invariant distortion model is:

$$r_1 = \frac{(1 + \sum_{i=0}^n k_i r^{2i}) r_0}{f}$$

Charts for distortion measurement: grid/disk patterns

Summation of Degradations

Example: Edge errors caused by chromatic aberration and distortion, sample from center to upper right corner



Vignetting

Especially for wide angle lenses, decreased aperture size for out of center image plane locations



Sensor, Image Signal Formation and Errors

Ideal Camera Signal

A n channel spectral sensor for every pixel would be nice:

$$\vec{d}_{Cam} = \begin{pmatrix} d_1 \\ \vdots \end{pmatrix} \sim \begin{pmatrix} \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) S_n(\lambda_i) \\ \vdots \end{pmatrix}$$

(λ_i) within range of 400..800 nm and a spectral resolution of $\Delta\lambda$ between 1..30 nm

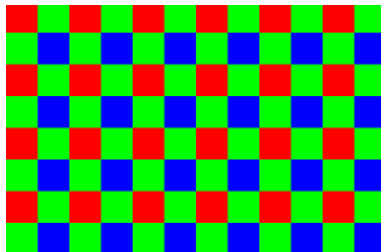
"RGB" Camera Signal

But n channels are hardly to produce, furthermore for human observers three channels might be enough for tristimulus registration, if sensitivity is comparable

$$\vec{C}_{Cam} = \begin{pmatrix} R_{Cam} \\ G_{Cam} \\ B_{Cam} \end{pmatrix} \sim \begin{pmatrix} \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) S_R(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) S_G(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) S_B(\lambda_i) \end{pmatrix}$$

BAYER Pattern

But a production problem exists, only side by side rgb sensor elements by filter arrays are easy to manufacture and inexpensive, BAYER 1975 for KODAK



Spectral Transmission and Sensitivity

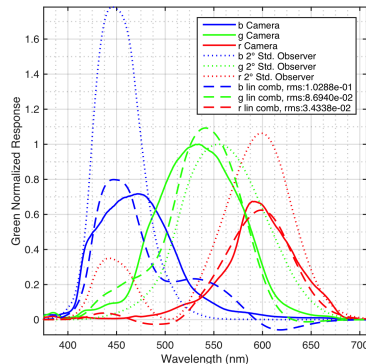
The overall spectral sensitivity S is formed by:

- Spectral transmission of filter $\tau_F(\lambda_i)$
- Spectral transmission of lens $\tau_L(\lambda_i)$
- Spectral transmission of sensor filter array $\tau_{FA}(\lambda_i)$ (with optional IR blocking)
- Spectral response of pn semiconductor $S_n(\lambda_i)$

$$d_n \sim \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) \tau_F(\lambda_i) \tau_L(\lambda_i) \tau_{FA}(\lambda_i) S_n(\lambda_i)$$

Overall Response Example

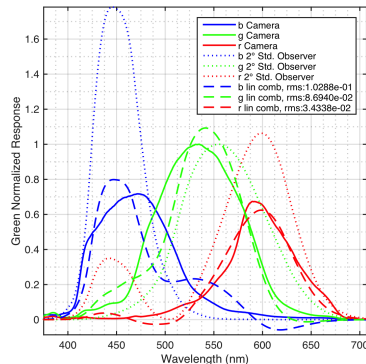
- A camera doesn't see colours!
- Non colourimetric, because transmissions and sensitivity are production limited and not equal to human observer response



Typical photo camera

Overall Response Example

- A camera doesn't see colours!
- The reconstruction of human observer tristimulus is called color management, and will be discussed in following lectures.



Typical photo camera

Other Errors

Sensor Structure Related Errors

- Spatial sampling errors by division of plane sensors
- Cross talk

Behind the Sensor

- Opto-Electronic Transfer Function (OETF), quantization
- Noise, dead pixel
- Dynamic of scene, bit depth
- Note: Discrepancy: lin vs. log encoding, WEBER-FECHNER Law

Capturing Errors

- Camera shake
- Motion blur
- Illumination instabilities
- Exposure time instabilities
- Shutter

A Camera System Sample

- Lens, sensor dimension
- Shutter, exposure times
- hands on...

Summary

V4 The Camera System as a Measure Device

Summary

- Simple camera model for geometric information
- A camera, its sensor and lenses are restricted by production technology
- Registered image pixels are influenced by a huge amount of errors
- Charts should be used to estimate system transfer functions by a calibration

Take Aways

Now you have a complete framework for geometric optics based image simulation.

Day 6

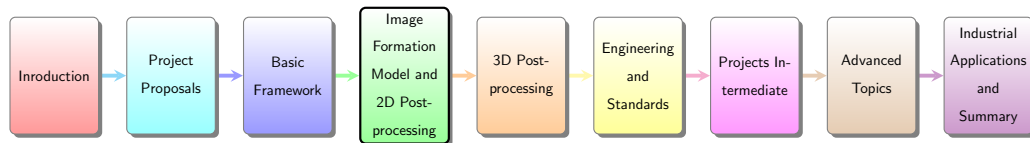
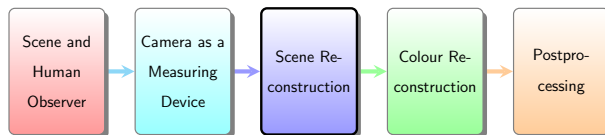


Image Formation and Processing

V5 Scene Reconstruction



Content I

- 6 V5 Scene Reconstruction
 - Noise and Defect Pixels Correction
 - Demosaicing
 - Lens Signal Degradation Correction
 - Lens Distortion Correction

- Learn to reconstruct the scene radiance, i.e. "Scene Referred" by eliminating related lens, sensor, electronic degradations
- Learn to reconstruct the geometry of ideal pinhole camera by eliminating related lens geometric distortions

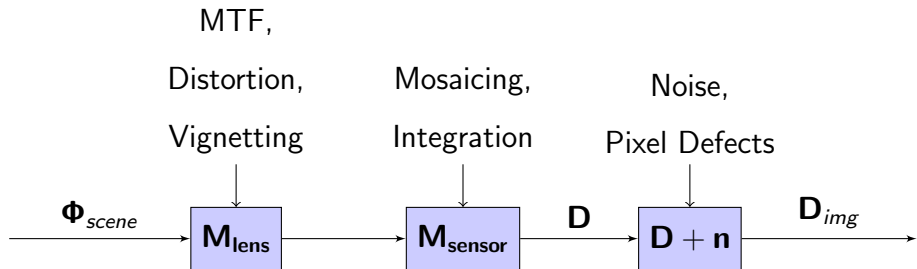
Image Formation and Scene Reconstruction

We try to reconstruct a geometric distortion free, scene radiance referred 2D image intensity distribution D in image plane from registered degraded camera image D_{cam} . For cameras: "Scene Referred" means really "Sensor Focal Plane Referred" General inverse problem of image reconstruction applying least square fit method:

$$\min_D ||D_{cam} - MD||_2^2$$

M is the system model which can be splitted in subsystems...

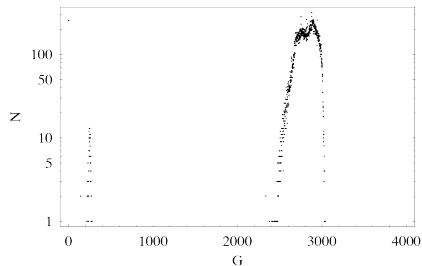
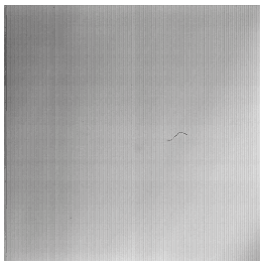
System Overview



Noise and Defect Pixels Correction

Recap Noise

A black noise image showing black frame pixels, inhomogeneous noise, dead pixels as well as particles on sensors surface:



Denoising

- Black noise reduction: Remove least significant bits
- White noise reduction: Apply GAUSS (Binomial) filter
- Noise: Mean image of images sequence
- Defect pixels: Interpolation (discussed later in demosaicing)

Denoising as a Minimization Problem

The model for noisy image formation:

$$D_{noise} = D + n$$

The variance of first and/or second derivatives can be used as an additional regularization for finding a solution for the inverse problem...

Denoising as a Minimization Problem

...if THIKONOV regularization based on L2 norm is used one can write

Problem:

$$\min_D (||D_{cam} - D||_2^2 + w_1 ||\nabla_h D||_2^2 + w_1 ||\nabla_v D||_2^2 + w_2 ||\nabla_h^2 D||_2^2 + w_2 ||\nabla_v^2 D||_2^2)$$

Solution (normal equation):

$$(E + w_1^2 \nabla_h^T \nabla_h + w_1^2 \nabla_v^T \nabla_v + w_2^2 (\nabla_h^2)^T \nabla_h^2 + w_2^2 (\nabla_v^2)^T \nabla_v^2) D = D_{cam}$$

This can be solved by inversion or numerically e.g. by conjugate gradient solver.

Disadvantage is the different quadratic of gradients.

Denoising as a Minimization Problem

A more appropriate method is total variation denoising using L1 norm, since the regularization term preserve sharp edges:

$$\min_D (||D_{cam} - D||_2^2 + w_1 ||\nabla_h D||_2 + w_1 ||\nabla_v D||_2 + w_2 ||\nabla_h D||_2^2 + w_2 ||\nabla_v D||_2)$$

Problem is that it can be solved only numerically, e.g. by alternative direction of multipliers method (ADMM). This method was further developed by RUDIN, OSHER and FATEMI and was applied to the first image of a black hole.

TV-L1 Denoising Sample

Lena (©Playboy)

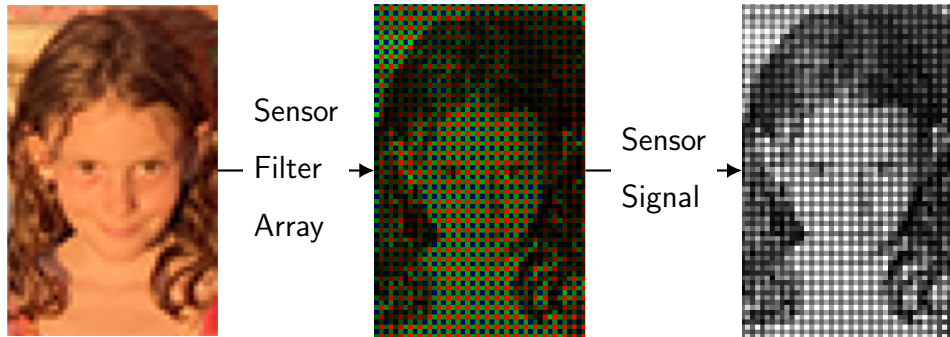


Note: Lena or Lena Söderberg aka Playboy 'Lenna Sjööblom' is one of the most used test images in image processing.

Demosaicing

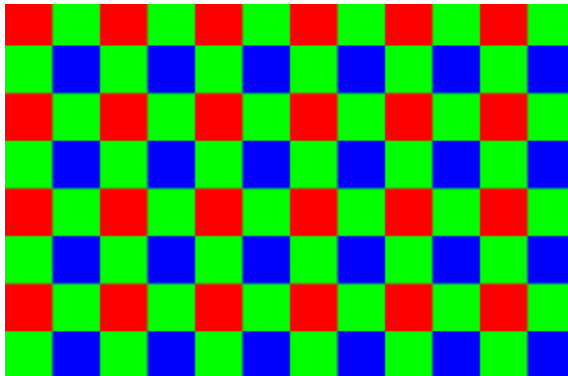
Recap: Mosaicing

Degradation by Subsampling

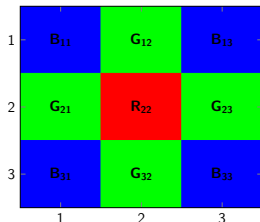


BAYER Pattern

BAYER Pattern

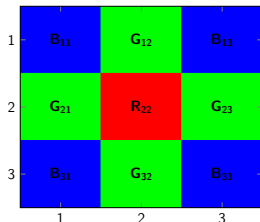


Demosaicing Problem



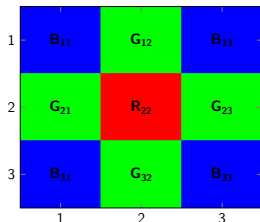
- Problem: Reconstruct missing samples, e.g.:
- $\mathbf{d}_{22} = (R_{22}, G_{22}?, B_{22}?)^T$
- Solution: reconstruct using known neighbourhood values

Demosaicing Problem



- 2D regular grid sensor
- Rectilinear or Cartesian grid
- Image resampling can be done by averaging or interpolation methods

Simple Averaging



$$d_{22} = \begin{pmatrix} R_{22} \\ \frac{1}{4}(G_{12} + G_{21} + G_{23} + G_{32}) \\ \frac{1}{4}(B_{11} + B_{13} + B_{31} + B_{33}) \end{pmatrix}$$

Advanced Averaging

Favour of lower gradients:

$$G_{22} = \begin{cases} \frac{G_{12} + G_{21} + G_{23} + G_{32}}{4}, & \Leftrightarrow |G_{12} - G_{32}| \wedge |G_{21} - G_{23}| < t \\ \frac{G_{21} + G_{23}}{2}, & \Leftrightarrow |G_{12} - G_{32}| > t \\ \frac{G_{12} + G_{32}}{2}, & \text{otherwise} \end{cases} \quad t := \text{threshold}$$

Do this same way for blue channel.

Demosaicing by Interpolation Polynomials

Common and general solution: interpolation by polynomials

$$d_{x,y} \approx \sum_{i=0}^n \sum_{j=0}^n a_{i,j} x^i y^j$$

The higher order allows a more smooth course , but it takes computation power.

Bilinear Interpolation

Bilinear interpolation (normalised grid coordinates):

$$d_{x,y} \approx \sum_{i=0}^1 \sum_{j=0}^1 a_{i,j} x^i y^j = a_{00} + a_{10}x + a_{01}y + a_{11}xy$$
$$= \begin{pmatrix} 1 & x & y & xy \end{pmatrix} \begin{pmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \end{pmatrix}$$

Bilinear Interpolation

Solution with coefficients:

$$\begin{pmatrix} d_{00} \\ d_{10} \\ d_{01} \\ d_{11} \end{pmatrix} = \begin{pmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_1 & y_2 & x_1 y_2 \\ 1 & x_2 & y_1 & x_2 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \end{pmatrix} \begin{pmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \end{pmatrix}$$

Bilinear Interpolation

It can be rewritten by following separation:

$$\begin{aligned}d(x, y) &\approx d(0, 0)(1 - x)(1 - y) + d(1, 0)x(1 - y) + d(0, 1)(1 - x)y + d(1, 1)xy \\&= \begin{pmatrix} 1 - x & x \end{pmatrix} \begin{pmatrix} d(0, 0) & d(0, 1) \\ d(1, 0) & d(1, 1) \end{pmatrix} \begin{pmatrix} 1 - y \\ y \end{pmatrix}\end{aligned}$$

Bicubic Interpolation

Bicubic polynomial for more smoothness:

$$d(x, y) \approx p(x, y) = \sum_{i=0}^3 \sum_{j=0}^3 a_{i,j} x^i y^j$$

Bicubic Interpolation

Coefficients for a unit square cartesian rectilinear grid:

$$p(0, 0) = a_{00}$$

$$p(1, 0) = a_{00} + a_{10} + a_{20} + a_{30}$$

$$p(0, 1) = a_{00} + a_{01} + a_{02} + a_{03}$$

$$p(1, 1) = \sum_{i=0}^1 \sum_{j=0}^1 a_{i,j} x^i y^j$$

Bicubic Interpolation

Derivative terms:

$$p'_x(x, y) = \sum_{i=1}^3 \sum_{j=0}^3 a_{i,j} i x^{i-1} y^j$$

$$p'_y(x, y) = \sum_{i=0}^3 \sum_{j=1}^3 a_{i,j} x^i j y^{j-1}$$

$$p'_{xy}(x, y) = \sum_{i=1}^3 \sum_{j=1}^3 a_{i,j} i x^{i-1} j y^{j-1}$$

Bicubic Interpolation

x derivatives in unit square grid:

$$p'_x(0, 0) = a_{10}$$

$$p'_x(1, 0) = a_{10} + 2a_{20} + 3a_{30}$$

$$p'_x(0, 1) = a_{10} + a_{11} + a_{12} + a_{13}$$

$$p'_x(1, 1) = \sum_{i=1}^1 \sum_{j=0}^1 a_{i,j} i$$

Bicubic Interpolation

y derivatives in unit square grid:

$$p'_y(0, 0) = a_{01}$$

$$p'_y(1, 0) = a_{01} + a_{11} + a_{21} + a_{31}$$

$$p'_y(0, 1) = a_{01} + 2a_{02} + 3a_{03}$$

$$p'_y(1, 1) = \sum_{i=0}^1 \sum_{j=1}^1 a_{i,j} j$$

Bicubic Interpolation

xy mixed derivatives in unit square grid:

$$p'_{xy}(0, 0) = a_{11}$$

$$p'_{xy}(1, 0) = a_{11} + 2a_{21} + 3a_{31}$$

$$p'_{xy}(0, 1) = a_{11} + 2a_{12} + 3a_{13}$$

$$p'_{xy}(1, 1) = \sum_{i=1}^1 \sum_{j=1}^1 a_{i,j} ij$$

Bicubic Interpolation

Equation system with solution:

$$\mathbf{M}_{\alpha} = \mathbf{M} \begin{pmatrix} a_{00} \\ a_{10} \\ a_{20} \\ a_{30} \\ a_{01} \\ a_{11} \\ a_{21} \\ a_{31} \\ a_{02} \\ a_{12} \\ a_{22} \\ a_{32} \\ a_{03} \\ a_{13} \\ a_{23} \\ a_{33} \end{pmatrix} = \begin{pmatrix} d(0,0) \\ d(1,0) \\ d(0,1) \\ d(1,1) \\ d'_x(0,0) \\ d'_x(1,0) \\ d'_x(0,1) \\ d'_x(1,1) \\ d'_y(0,0) \\ d'_y(1,0) \\ d'_y(0,1) \\ d'_y(1,1) \\ d'_{xy}(0,0) \\ d'_{xy}(1,0) \\ d'_{xy}(0,1) \\ d'_{xy}(1,1) \end{pmatrix} = \mathbf{d}$$

Bicubic Interpolation

Equation system with solution:

$$\mathbf{M}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 & -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 3 & 0 & 0 & -2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 & 1 & 1 & 0 & 0 \\ -3 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & -1 & 0 \\ 9 & -9 & -9 & 9 & 6 & 3 & -6 & -3 & 6 & -6 & 3 & -3 & 4 & 2 & 2 & 1 \\ -6 & 6 & 6 & -6 & -3 & -3 & 3 & 3 & -4 & 4 & -2 & 2 & -2 & -2 & -1 & -1 \\ 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ -6 & 6 & 6 & -6 & -4 & -2 & 4 & 2 & -3 & 3 & -3 & 3 & -2 & -1 & -2 & -1 \\ 4 & -4 & -4 & 4 & 2 & 2 & -2 & -2 & 2 & -2 & 2 & -2 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Note: Dense vs. sparse matrices in programmers world...

Bicubic Interpolation

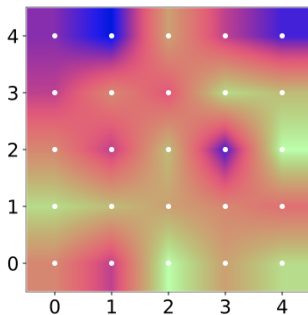
Alternative simplified application of solution using separability:

$$d(x, y) = \begin{pmatrix} 1 & x & x^2 & x^3 \end{pmatrix} \begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 1 \\ y \\ y^2 \\ y^3 \end{pmatrix}$$

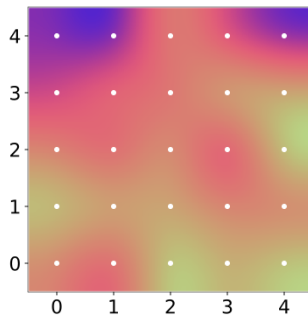
Interpolation Comparison

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bilinear



bicubic



Interpolation Comparison

- Higher Order better preserve local edges
- Negative lobes on the kernel, haloing/clipping
- It increases acutance (apparent sharpness), subjective desirable.
- Operators also be used in general image scaling, not only demosaicing

Demosaicing using Regularization

RG/GB BAYER Mosaicing as algebraic expression with selector matrix M_{sel} :

$D_{cam} = M_{sel}D$, image reshaped as $n \times M$ row/one column vector, e.g. M_{sel} for a 2×4 RGB image:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Demosaicing as a Minimization Problem

We can combine the denoising with the demosaicing, e.g. for THIKONOV regularisation:

$$\min_D ||D_{cam} - M_{sel}D||_2^2 + w_1 ||\nabla_h D||_2^2 + w_1 ||\nabla_v D||_2^2 + w_2 ||\nabla_h^2 D||_2^2 + w_2 ||\nabla_v^2 D||_2^2$$

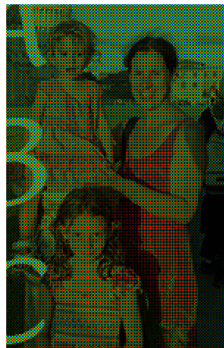
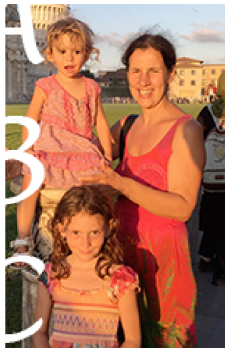
with solution expressed as normal equation:

$$(M_{sel}^T M_{sel} + w_1^2 \nabla_h^T \nabla_h + w_1^2 \nabla_v^T \nabla_v + w_2^2 (\nabla_h^2)^T \nabla_h^2 + w_2^2 (\nabla_v^2)^T \nabla_v^2) D = M_{sel}^T D_{cam}$$

It can be done similar for TV-L1 demosaicing and denoising.

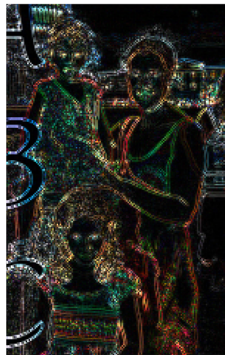
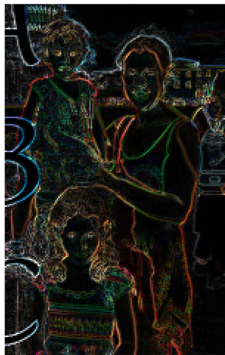
Demosaicing THIKONOV Regularisation Example

Ground Truth/Subsampled(RGB)/Subsampled(Gray) as Input



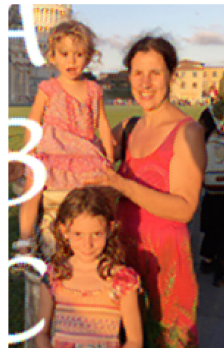
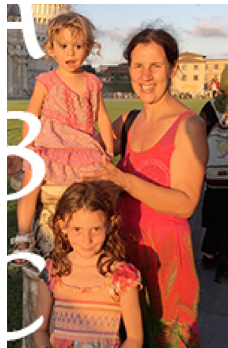
Demosaicing THIKONOV Regularisation Example

First/Second Derivatives



Demosaicing THIKONOV Regularisation Example

Ground Truth vs. Estimate (PSNR=60.5 dB)



Demosaicing using Correlations

Till now we did it channel-wise, but what about using image region/channels correlations?

- Spatial correlation: similar color values in pixel neighbourhood
- Spectral correlation: dependency between the pixel values of different image channels

Demosaicing using Correlations

Methods/Algorithms:

- Variable Number of Gradients (VNG): interpolation computes gradients near the pixel of interest and uses the lower gradients (representing smoother and more similar parts of the image)
- Pixel Grouping (PPG): uses assumptions about natural scenery

Demosaicing using Correlations

Methods/Algorithms:

- Adaptive Homogeneity-Directed (AHD): interpolation selects the direction of interpolation so as to maximize a homogeneity metric
- Aliasing Minimization and Zipper Elimination (AMaZE) MARTINEC
- Deep learning based

Lens Signal Degradation Correction

Vignetting

Flat Field Correction

- $d = m \frac{d_{cam} - d_b}{d_f - d_b}$
- $m = \overline{(D_f - D_b)}$
- m : mean value of related image
- Without noise: $d = \frac{\overline{(D_f)}}{d_f} d_{cam}$



Flat Field Correction

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WIENER filter (1940s)

Reduce degradation and noise in images degraded by following model

$$D_{cam}(\mathbf{r}) = (B * D)(\mathbf{r}) + N(\mathbf{r}), \text{ with}$$

- D degradation free image
- known blurring shift-invariant function $B(\mathbf{r})$ and
- additive noise $N(\mathbf{r})$

WIENER filter (1940s)

The Fourier transform in the frequency domain of this degraded image $D_{cam}(\mathbf{r})$ is

$$\hat{D}_{cam}(\mathbf{f}) = \mathcal{F}(D_{cam})(\mathbf{f}) = \hat{B}(\mathbf{f})\hat{D}(\mathbf{f}) + \hat{N}(\mathbf{f})$$

WIENER filter

Problem: find an appropriate filter $W(\mathbf{f})$ such that $W(\mathbf{f})\hat{D}_{cam}(\mathbf{f})$ is as much close as possible to the Fourier transform of the original signal

$\hat{D}(\mathbf{f}) = \mathcal{F}(D)(\mathbf{f})$, i.e. we want to minimize the quantity

$$W(\mathbf{f})\hat{D}_{cam}(\mathbf{f}) - \hat{D}(\mathbf{f}) = W(\mathbf{f})(\hat{B}(\mathbf{f})\hat{D}(\mathbf{f}) + \hat{N}(\mathbf{f})) - \hat{D}(\mathbf{f})$$

WIENER filter

Solution:

$$W(\mathbf{f}) = \frac{\hat{B}^*(\mathbf{f})}{|\hat{B}(\mathbf{f})|^2 + |\hat{N}(\mathbf{f})|^2 / |\hat{D}(\mathbf{f})|^2}$$

It is the WIENER filter function.

Applying it, gives the filtered signal $\mathcal{F}^{-1}(W\hat{D}_{cam})(\mathbf{r})$. It gives more importance to frequencies with higher SNR.

WIENER filter

In absence of noise it can be simplified as

$$\begin{aligned}
 W(f) &= \frac{\hat{B}^*(f)}{|\hat{B}(f)|^2 + |\hat{N}(f)|^2 / |\hat{D}(f)|^2} \\
 &= \frac{\hat{B}^*(f)}{|\hat{B}(f)|^2} = \frac{\hat{B}(f)}{\hat{B}(f)} \frac{\hat{B}^*(f)}{|\hat{B}(f)|^2} \\
 &= \frac{1}{\hat{B}(f)}
 \end{aligned}$$

WIENER filter

The Wiener filter function requires:

- The blurring function (can be estimated by PSF/OTF)
- The noise (can be estimated from noise image)
- The original signal (not given)...

WIENER filter

Power spectra of original scene need not to be known exactly:

- Most signals of the same class have fairly similar power spectra
- WIENER filter is insensitive to small variations in the original signal power spectrum

We can estimate the original signal power spectrum using a representative of the class of signals being filtered.

WIENER Filter Example

Expected/Captured Chart Image



The PSF coefficients can be estimated by fitting the ideal sine pattern to the PSF blurred/convoluted captured chart:: $\min_{h_h} \tilde{D} - \tilde{h}_h \tilde{D}_{cam}$

Remember that an one dimensional sine pattern chart is enough, the 2D kernel then is $h = h_h h_h^T$.

WIENER Filter Example

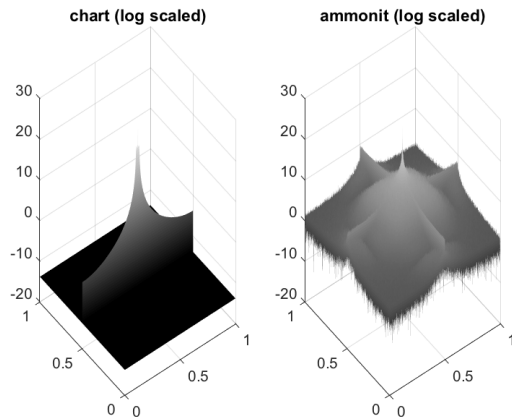
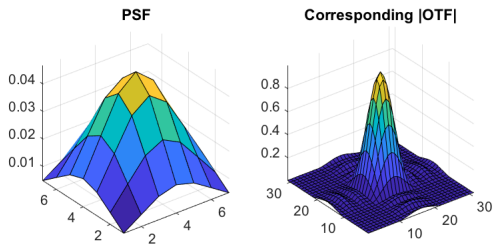
Original Sample Image from an Ammonit



WIENER Filter Example

Spectra of sine wave chart and ammonit

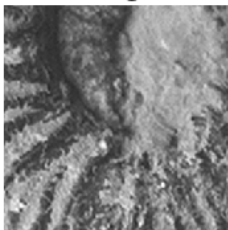
The resulting PSF and its FOURIER transformation OTF



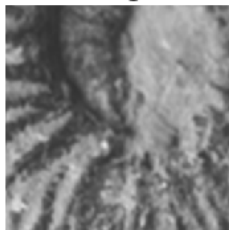
WIENER Filter Example

PSF degradation but without noise(origin/registered/reconstructed)

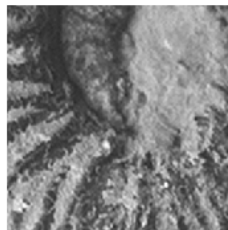
org



reg



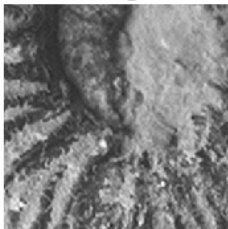
reconstructed



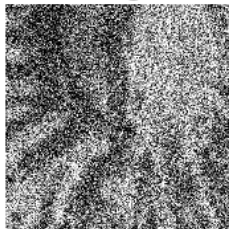
WIENER Filter Example

PSF/noise degradation(origin/registered/reconstructed)

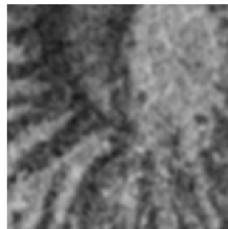
org



reg



reconstructed



WIENER Filter

WIENER filter can remove different convolution artefacts, if you can estimate the convolution kernel.

- Different kind of noise: photon, sensor, quantization
- Camera motion blurring...

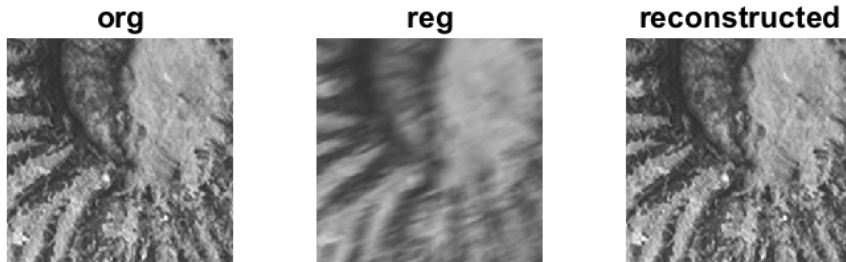
WIENER Filter Example

Camera motion blurring kernel, a combination of PSF and translation during exposition, e.g.:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.012247 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.011618 & 0.028669 & 0.04572 & 0.036937 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0073123 & 0.024363 & 0.041414 & 0.041242 & 0.024191 & 0.0071403 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0030066 & 0.020058 & 0.037109 & 0.045548 & 0.028497 & 0.011446 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.015752 & 0.032803 & 0.049854 & 0.032803 & 0.015752 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.011446 & 0.028497 & 0.045548 & 0.037109 & 0.020058 & 0.0030066 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0071403 & 0.024191 & 0.041242 & 0.041414 & 0.024363 & 0.0073123 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.036937 & 0.04572 & 0.028669 & 0.011618 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.012247 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

WIENER Filter Example

Camera motion blurring and restored image, with kernel from slide before:



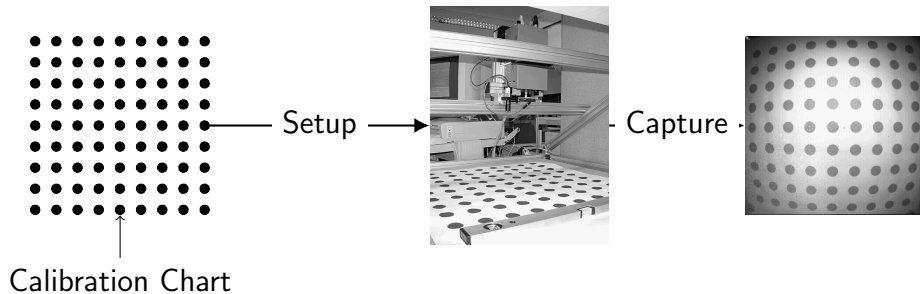
Lens Distortion Correction

Rectification

Now we have correct pixel intensities, but we have to correct geometric degradation too

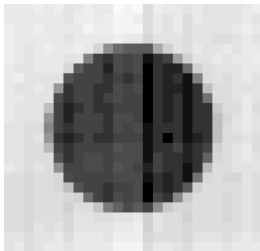
- Important for location measurements
- Captured chart with known geometry model
- Find chart patch locations
- Find and apply inverse

Rectification

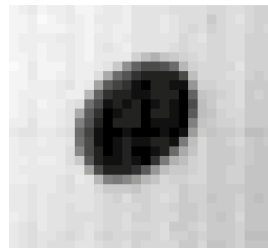


Rectification

E.g. find center and locations for disk shapes using center of gravity algorithms and fitting rotated ellipsis equation:



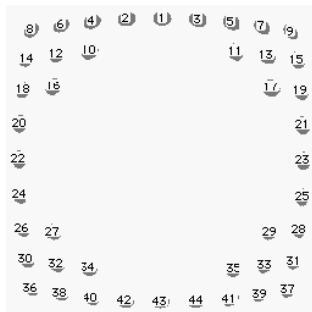
Disk



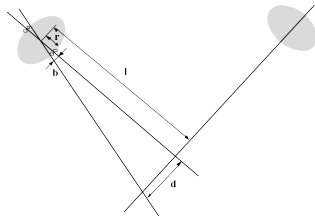
Distorted Ellipse

Rectification

Find Center



Subset for finding center

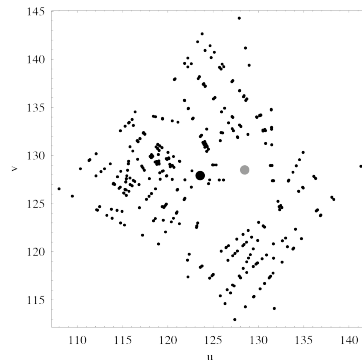


Center and its error for two ellipses

Rectification

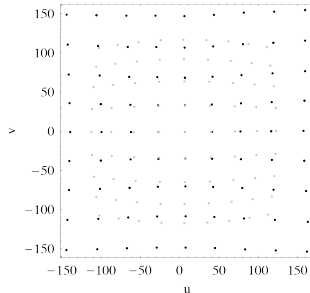
Find center results

- Center point cloud
- Found center (black disk)
- Ideal center (gray disk)

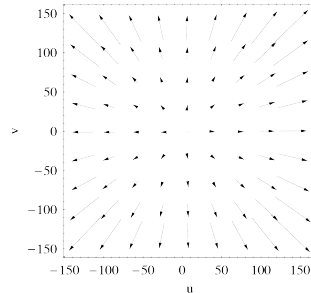


Rectification

Find Distortion



Expected/Registered Locations

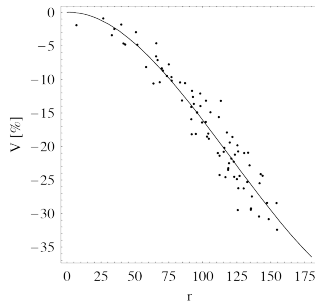


Shift Vector Field

Rectification

Find Distortion Polynomial $r_1 = \frac{1}{f}(1 + \sum_{i=0}^n k_i r^{2i})r_0$

- All point cloud
- Fit
- Inverse with inverse lookup



Least Square Fitted p3 Polynomial

Summary

V5 Scene Reconstruction

Summary

- Signal reconstruction removing sensor/lens artefacts
- Ideal pinhole location reconstruction

Take Aways

At best now we have an approximation of degradation free reconstructed intensity distribution. But this original scene radiance reconstruction is more a sensor plane referred one!

Day 7

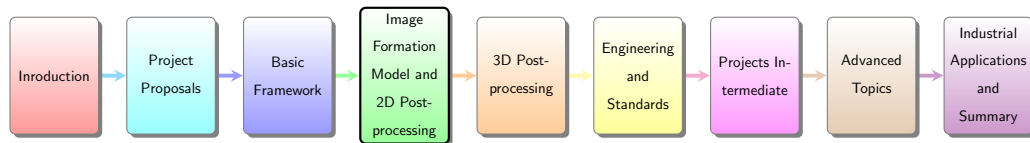
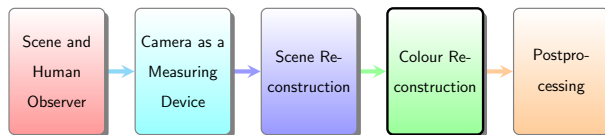


Image Formation and Processing

V6 Colour Reconstruction



Content I

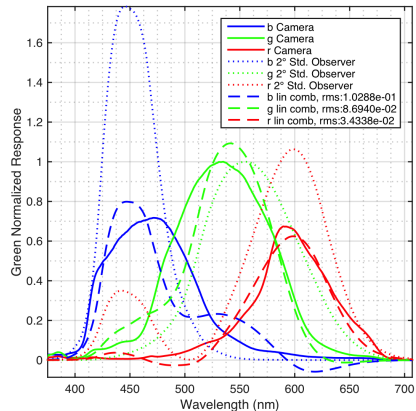
- 7 V6 Colour Reconstruction
 - Human Observer vs. Camera
 - Camera System Spectral Response Estimation
 - Camera Characterisation
 - Perception Based Camera Characterization

- The difference between human and camera observers
- Estimate camera response
- Learn to reconstruct the scene colour appearance from image data, i.e. camera characterization

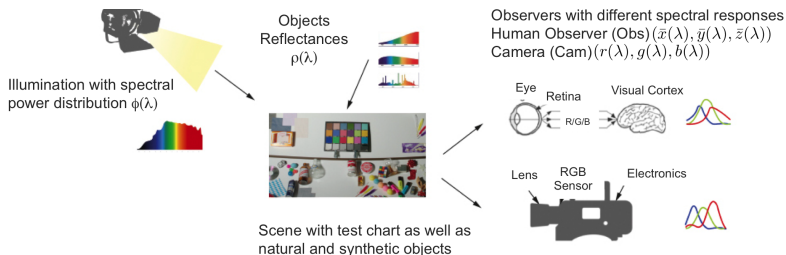
Human Observer vs. Camera

LUTHER Condition

- Luther condition (Luther 1927): the sensitivity of any trichromatic system must be a linear combination of the colour matching functions, in order to obtain same tristimulus
- This is in general not fulfilled by cameras, for technical reasons, e.g. caused by filter pigment spectral absorption curves
- Right image shows the normalized responses of a Canon 5D Mark III camera, the CIE 2° standard observer and best fitting camera linear combination of standard observer



Human Eye vs. Technical Observer



- The registered camera "colour" differs from human observed colour

$$\vec{C}_{Obs} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \sim \begin{pmatrix} \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) \bar{x}(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) \bar{y}(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) \bar{z}(\lambda_i) \end{pmatrix} \neq \begin{pmatrix} \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) r(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) g(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) b(\lambda_i) \end{pmatrix} \sim \begin{pmatrix} R_{Cam} \\ G_{Cam} \\ B_{Cam} \end{pmatrix} = \vec{C}_{Cam}$$

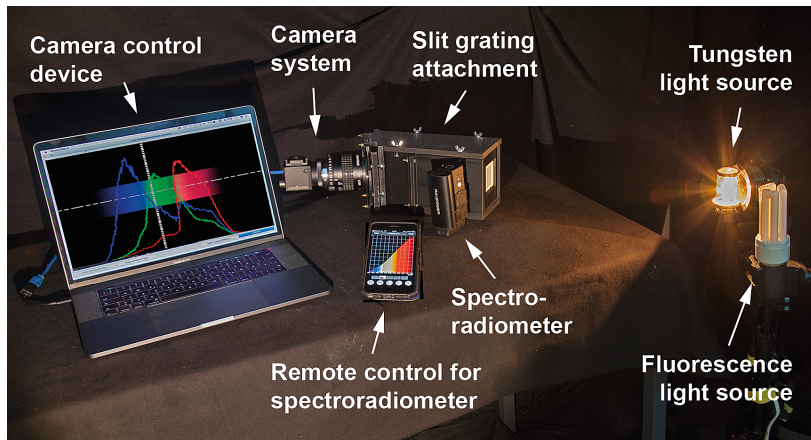
Camera System Spectral Response Estimation

Camera Response Estimation

- Classical approach uses monochromator
- Image processing based: "Fast Camera Spectral Responsivity Measurement Using Chromaticity Error Minimisation" by KARGE et al. 2018, improvement of Open Film Tools (OFTex)

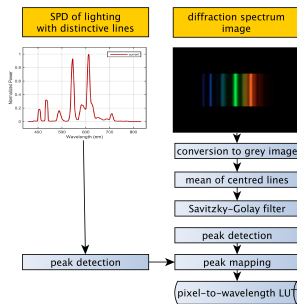
OFTex Setup

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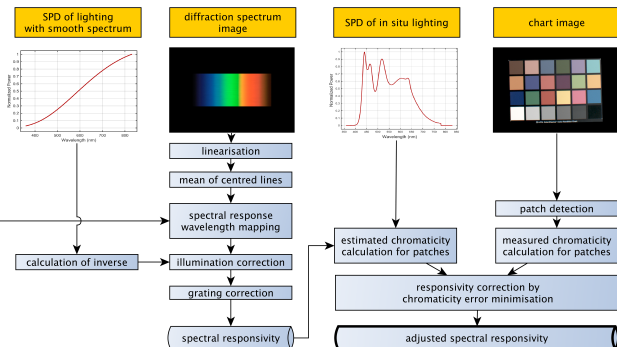
OFTex Algorithm Overview

Pixel Coordinate-to-Wavelength Mapping



laboratory setup

Responsivity Estimation Using Chromaticity Error Minimisation

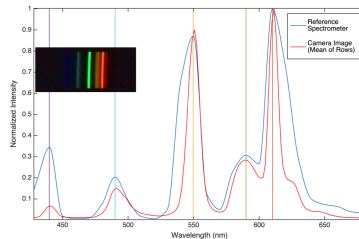


in situ setup

OFTex Algorithm I

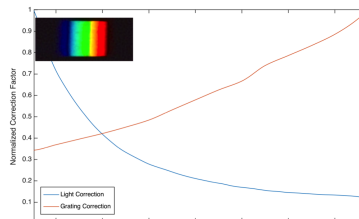
(1) Geometric Calibration with Line Lighting

- OSRAM Energy Saving light source
- Estimating the pixel to wavelength mapping function by using distinct lines (spectrometer reference measurement)
- Biggest challenge: reducing interference effects



(2) Radiometric Calibration with Tungsten Lighting

- Correction for grating efficiency (provided by manufacturer) and SPD of illumination (spectrometer reference measurement)
- Biggest challenge: strong slope in blue range

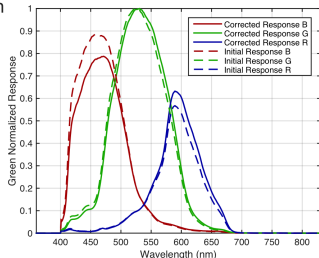


OFTex Algorithm II

(3) Adding an extended correction algorithm using an additional testchart image, in order to get parameters a , b of a linear correction function

$$\min_{a,b} \| \mathbf{C}_{Cam,G}^T - \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) (a(\lambda_i) - b) g(\lambda_i) \|$$

- For a Canon 5D Mark III the response before and after correction is shown below, right table shows the final green normalized RGB pixel value deviation



	Red CAM	Green CAM	Blue CAM
dark skin	0.9672	1	1.0312
light skin	0.9345	1	1.0149
blue sky	0.9426	1	1.0588
foliage	0.9620	1	1.0130
blue flower	0.9343	1	1.0631
bluish green	0.9541	1	1.0293
orange	0.9760	1	1.0161
purplish blue	0.9111	1	1.0550
moderate red	0.9594	1	1.0364
purple	0.9310	1	1.1123
yellow green	0.9690	1	1.0155
orange yellow	0.9747	1	1.0146
blue	0.8692	1	1.0522
green	0.9064	1	0.9641
red	1.0118	1	1.0274
yellow	0.9513	1	1.0390
magenta	0.9337	1	1.0804
cyan	0.9040	1	1.0324
white 9.5 (.05 D)	0.9334	1	1.0625
neutral 8 (.23 D)	0.9448	1	1.0496
neutral 6.5 (.44 D)	0.9485	1	1.0487
neutral 5 (.70 D)	0.9508	1	1.0412
neutral 3.5 (1.05 D)	0.9566	1	1.0242
black 2 (1.5 D)	0.9417	1	1.0567
Mean	0.9445	1	1.0391
Sigma	0.0287	0	0.0285

OFTex Algorithm III

- Lightness independent, chromaticity based
- $\mathbf{c}_{\text{Cam}}$ are registered camera chromaticities for given object reflectances ρ and illumination Φ taken from test chart image
- the estimated response S will be corrected by a function $f_{\text{cor}}(\mathbf{a})$ and white balancing $\mathbf{w}$

$$f_{\text{cor}}(\lambda) = a_0 + 0.01a_1 \frac{\lambda - \bar{\lambda}}{\Delta\lambda} + 0.0001a_2 \left(\frac{\lambda - \bar{\lambda}}{\Delta\lambda} \right)^2$$

$$\mathbf{a} = (a_0, a_1, a_2)^T$$

$$\mathbf{w} = (w_R, w_G, w_B)^T$$

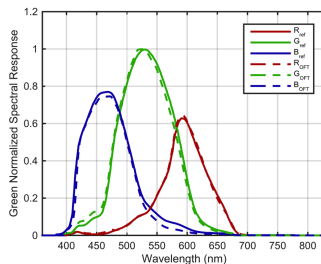
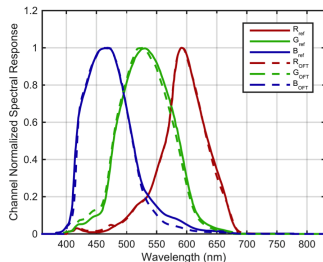
$$\mathbf{c} = (r, g, b)^T, r = \frac{R}{R+G+B}, g = \frac{G}{R+G+B}, b = \frac{B}{R+G+B}$$

- $\mathbf{a}$ and $\mathbf{w}$ are determined by error minimization:

$$\min_{\mathbf{a}, \mathbf{w}} \left\| \mathbf{c}_{\text{Cam}} - \frac{w_k \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) f_{\text{cor}}(\lambda_i) \tilde{S}_k(\lambda_i)}{\sum_{k=R,G,B} w_k \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) f_{\text{cor}}(\lambda_i) \tilde{S}_k(\lambda_i)} \right\|_{p=2}$$

OFTex Measurement Results

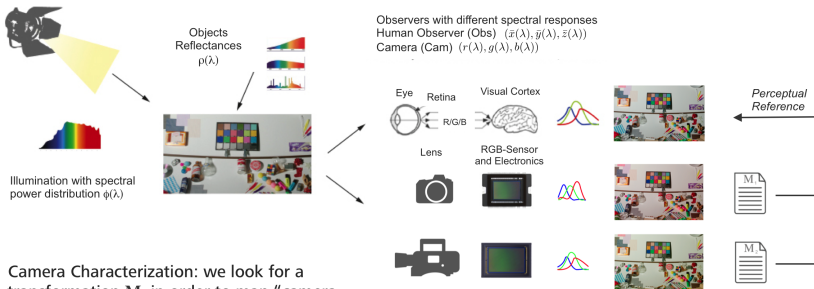
- Canon - 5D Mark III, lens EF 24-70mm f/2.8L II USM , Reference: [MG2015]



[MG2015] Jon S McElvain and Walter Gish. "Cinematic camera emulation using two-dimensional color transforms." In: Digital Photography. 2015, p. 940401.

Camera Characterisation

Camera Characterization

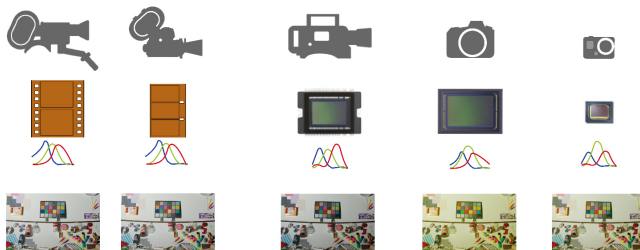


- Camera Characterization: we look for a transformation $\mathbf{M}$, in order to map "camera colour" into human colour

$$\vec{C}_{Obs} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \sim \begin{pmatrix} \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) \bar{x}(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) \bar{y}(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) \bar{z}(\lambda_i) \end{pmatrix} \approx \mathbf{M} \begin{pmatrix} \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) r(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) g(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) b(\lambda_i) \end{pmatrix} \sim \begin{pmatrix} R_{Cam} \\ G_{Cam} \\ B_{Cam} \end{pmatrix} = \vec{C}_{Cam}$$

Motivation

Problem: Different spectral responses, especially for digital cameras

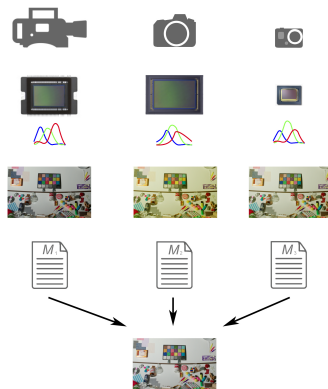


Note: Before digital imaging, same silver halid emulsions could be used in different cameras

Motivation

Camera Profile:

- Container file for transformation matrix...
- File formats: ICC, IDT, DCP...



Camera Characterization aka Profile Creation

Problem: Finding matrix **M**

$$\vec{C}_{Obs} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \sim \begin{pmatrix} \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) \bar{x}(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) \bar{y}(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) \bar{z}(\lambda_i) \end{pmatrix} \approx \mathbf{M} \begin{pmatrix} \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) r(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) g(\lambda_i) \\ \sum_{i=1}^n \phi(\lambda_i) \rho(\lambda_i) b(\lambda_i) \end{pmatrix} \sim \begin{pmatrix} R_{Cam} \\ G_{Cam} \\ B_{Cam} \end{pmatrix} = \vec{C}_{Cam}$$

Solution

- Basic approach: a set of $m > 3$ colour tristimuli

$$\mathbf{C}_{Obs}^T = \begin{pmatrix} X_1 & Y_1 & Z_1 \\ \vdots & & \\ X_m & Y_m & Z_m \end{pmatrix} \approx \begin{pmatrix} R_1 & G_1 & B_1 \\ \vdots & & \\ R_m & G_m & B_m \end{pmatrix} \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} = \mathbf{C}_{Cam}^T \mathbf{M}^T$$

- Problem is as follows, whereas $\mathbf{M}$ is the 3×3 transformation to be found (transpose cause it is convenient that colour is a column vector, Camera data is linearized before)

$$\min_M \| \mathbf{C}_{Obs}^T - \mathbf{C}_{Cam}^T \mathbf{M}^T \|$$

- The colour transformation matrix is then be used for colour correction of images
- The quantitative evaluation of the colour correction quality after applying the transformation can be done by using CIE Delta E or Delta 2000 for selected objects

Input Data Parameter Set

- In order to having reference and camera tristimuli we need the spectral characteristics (often not in a analytical closed form, but measured samples):
 - A set of object reflectances as training spectra (e.g. test charts, ISO-SOCS), and a method defining a meaningful subset for optimization
 - An illumination spectrum (not given by Manufacturer), in situ measured, SPD of comparable lighting, or CCT equivalent SPD
 - The reference observer (CIE defined) and camera response (not given by manufacturer), to be measured before

The Image Flair

Choosing a certain subset of object reflectance influences the color appearance
flair:

- Portrait Mode, Landscape Mode ...
- The secret of the manufacturer

The Lens Flair

Technology
Arts Sciences
TH Köln

Sometimes camera men do not like correct reproduction for certain lenses:
Old Bausch & Lomb vs. Zeiss Ultra Prime Cine Lens



Zeiss Ultra Prime



Bausch & Lomb

Matrix Based Camera Characterization Methods

- First summary and evaluation of simple least square (Hubel 1997) LCC
- Usage of constrained least square: Colorimetric white point preserving (Finlayson 1997) WPPLCC
 - Eye is sensitive to white/neutral shift, white must be preserved
 - Requires selection of neutral/white tristimulus w for reference and camera
 - Preprocessing step: normalize reference and camera data tristimulus values by neutral tristimulus component values, i.e. in particular the white tristimuli are vectors of ones
 - Constraint: $w_{\text{ref}} = (1 \ 1 \ 1)^T = M (1 \ 1 \ 1)^T = M w_{\text{Cam}}$

Matrix Based Camera Characterization Methods

- Better characterization with non linear polynomials by using other then linear combinations of basic components, i.e. estimating a $n \times 3$ colour correction matrix ($n > 3$):

$$\min_M \| \mathbf{C}_{Obs}^T - \tilde{\mathbf{C}}_{Cam}^T \mathbf{M}^T \|$$

- Higher Order Polynomials (Hong 2001) PCC, disadvantage is nonlinear lightness scaling, e.g.:

$$\tilde{\mathbf{C}}_{Cam,i}^T = (R_i \quad G_i \quad B_i \quad R_i G_i B_i \quad 1)$$

$$\tilde{\mathbf{C}}_{Cam,i}^T = (R_i \quad G_i \quad B_i \quad R_i G_i \quad R_i B_i \quad B_i G_i \quad R_i^2 \quad G_i^2 \quad B_i^2)$$

- Root Polynomials (Finlayson 2011) RPCC, enables lightness ratio preserving, e.g.

$$\tilde{\mathbf{C}}_{Cam,i}^T = (R_i \quad G_i \quad B_i \quad \sqrt{R_i G_i} \quad \sqrt{R_i B_i} \quad \sqrt{B_i G_i})$$

- Evaluation methods and results:
 - Leave One Out Method
 - some transformation have better delta E but were worse in perceptual domain
- Side not: Matlab pitfall data ordering, inconsistent API

Hue Plane Preserving Colour Correction

- Described in 2016 Mackiewicz, Andersen, Finlayson; Method for hue plane preserving color correction* and 2015 Hue Plane Preserving Colour Correction using Constrained Least Square Regression (NHPPCC)
- Previous work 2005 Andersen, Hardeberg; Colorimetric characterization of digital cameras preserving hue planes (HPPLCC)
- New idea: not only one but a set of matrices, everyone responsible for different chromaticity range

*(*figures, tables on next slides taken by this reference)*

(N)HPPCC – Hue Planes

- Preserving hue planes, i.e. the dominant wavelength for colours are not modified after applying the transformation

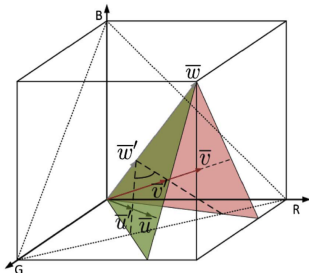


Fig. 3. Visualization of the convex cone spanned by the two hue planes in the RGB unit cube. A hue plane is spanned by the neutral vector $\bar{w}$ and a chromatic color ($\bar{u}$ or $\bar{v}$). These three vectors intersect the chromaticity plane (dotted triangle) at $\bar{w}'$, $\bar{u}'$, and $\bar{v}'$. The hue planes intersect the chromaticity plane in hue lines (dashed lines).

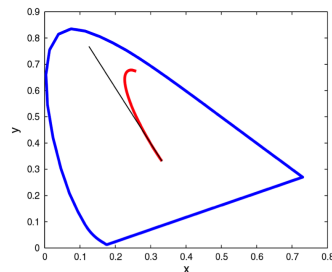


Fig. 4. CIE xy chromaticity diagram and a sample hue plane distortion resulting from the root-polynomial color correction of degree two (red) [21] and nonmodified hue line (black).

(N)HPPCC – Sample Selection and Preprocessing

- Tristimuli sample selection with least susceptibility to noise (whereas n is number of selected samples)
 - Open question: what does it mean: choose RGB sample with least susceptibility to noise
- Samples normalization to white
- Compute „camera chromaticities“ for every camera sample $i = 1..n$:

$$r_i = R_i / (R_i + G_i + B_i), g_i = G_i / (R_i + G_i + B_i), b_i = B_i / (R_i + G_i + B_i)$$

(N)HPPCC – Sample Selection and Preprocessing

- Compute angle θ_i in „camera chromaticity“ plane for every camera sample and sorting data by angle (transformation of cartesian „camera chromaticity“ r/g coordinates into polar coordinates and additional translation of camera white into center)

$$\theta_i = \begin{cases} 0 & \text{if } r_i = g_i = \frac{1}{3}, \\ \frac{\pi}{2} & \text{if } r_i = \frac{1}{3} \wedge g_i > \frac{1}{3}, \\ \frac{3\pi}{2} & \text{if } r_i = \frac{1}{3} \wedge g_i < \frac{1}{3}, \\ \arctan \frac{g_i - \frac{1}{3}}{r_i - \frac{1}{3}} + m\pi & \text{otherwise,} \end{cases}$$

where

$$m = \begin{cases} 0 & \text{if } r_i \geq \frac{1}{3} \wedge g_i \geq \frac{1}{3}, \\ 1 & \text{if } g_i < \frac{1}{3}, \\ 2 & \text{otherwise,} \end{cases}$$

(N)HPPCC – Split into Hue Regions

- Split camera chromaticity plane into K slices/hue regions, whereas certain selected sample tristimulus for the k^{th} slice q_k is situated at the slice lower boundary and goes to the upper boundary's tristimulus q_{k+1} which defines the start of next $k^{\text{th}+1}$ slice, if $k=K$ then $k+1$ is set to $k=1$ (wraparound); "camera chromaticity" for white w is defined as $(1/3 \ 1/3 \ 1/3)^T$
 - Open question: Missing explanation how the boundaries samples are defined, maybe equidistantly or same number of samples per region
- First HPPCC: One equation system, respectively colour correction matrix $\mathbf{T}_k$ per k^{th} hue region, p is the corresponding reference tristimulus

$$\begin{bmatrix} \bar{q}_k^T \\ \bar{q}_{k+1}^T \\ \bar{w}^T \end{bmatrix} \mathbf{T}_k = \begin{bmatrix} \bar{p}_k^T \\ \bar{p}_{k+1}^T \\ \bar{p}_w^T \end{bmatrix} \quad \text{for } k = 1 \dots k-1$$

$$\begin{bmatrix} \bar{q}_k^T \\ \bar{q}_1^T \\ \bar{w}^T \end{bmatrix} \mathbf{T}_k = \begin{bmatrix} \bar{p}_k^T \\ \bar{p}_1^T \\ \bar{p}_w^T \end{bmatrix} \quad \text{for } k = k,$$

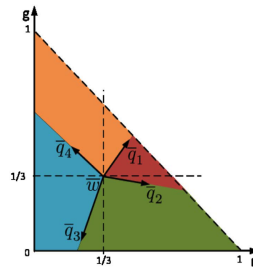


Fig. 5. Construction of hue regions in the rg chromaticity space.

NHPPCC – Minimization Problem

- $\mathbf{Q}_k$ ($N_k \times 3$ matrices), are all camera samples of k^{th} hue slice, $\mathbf{t}_k$ the n^{th} column of transformation $\mathbf{T}_k$, and $\mathbf{x}_k$ the corresponding reference values of the n^{th} tristimulus component, then the optimization problem is

$$\underset{\tilde{\mathbf{t}}_1 \dots \tilde{\mathbf{t}}_K}{\text{minimize}} \sum_{k=1}^K \|\mathbf{Q}_k \tilde{\mathbf{t}}_k - \tilde{\mathbf{x}}_k\|$$

- Or rewritten as

$$\underset{\tilde{\mathbf{T}}}{\text{minimize}} \|\mathbf{A} \tilde{\mathbf{T}} - \tilde{\mathbf{X}}\|,$$

where $\mathbf{A}$ is a nonsquare ($N \times 3K$) block diagonal matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{Q}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{Q}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{Q}_K \end{bmatrix},$$

$\tilde{\mathbf{T}}$ is a $3K$ -vector $\tilde{\mathbf{T}} = (\tilde{\mathbf{t}}_1^T, \dots, \tilde{\mathbf{t}}_K^T)^T$, and $\tilde{\mathbf{X}}$ is an N -vector $\tilde{\mathbf{X}} = (\tilde{\mathbf{x}}_1^T, \dots, \tilde{\mathbf{x}}_K^T)^T$.

NHPPCC – Set Up of Constraints

- Constrained: C0 continuity at hue regions boundaries, same white point for all regions matching same reference value

$$\tilde{q}_k^T \tilde{t}_k = \tilde{q}_k^T \tilde{t}_{k+1}, \quad k = 1, \dots, K-1,$$

$$\tilde{q}_k^T \tilde{t}_k = \tilde{q}_k^T \tilde{t}_1, \quad k = K,$$

$$\tilde{w}^T \tilde{t}_1 = \tilde{w}^T \tilde{t}_2 = \dots = \tilde{w}^T \tilde{t}_K = x_w,$$

- Constraint in Matrix form

$$\mathbf{C} \tilde{\mathbf{T}} = \tilde{\mathbf{b}},$$

where $\mathbf{C}$ is a $2K \times 3K$ matrix:

$$\mathbf{C} = \begin{bmatrix} \tilde{q}_1^T & -\tilde{q}_1^T & 0 & \dots & 0 \\ 0 & \tilde{q}_2^T & -\tilde{q}_2^T & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \tilde{q}_{K-1}^T & -\tilde{q}_{K-1}^T \\ -\tilde{q}_K^T & 0 & \dots & 0 & \tilde{q}_K^T \\ \tilde{w}^T & -\tilde{w}^T & 0 & \dots & 0 \\ 0 & \tilde{w}^T & -\tilde{w}^T & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \tilde{w}^T & -\tilde{w}^T \\ \tilde{w}^T & \dots & 0 & \dots & 0 \end{bmatrix},$$

and

$$\tilde{\mathbf{b}} = \begin{bmatrix} \tilde{\mathbf{0}} \\ x_w \end{bmatrix}.$$

NHPPCC – Whole Optimization Problem and Results

- Whole optimization problem for n^{th} Component of colour transformation matrix

$$\underset{\bar{T}}{\text{minimize}} \quad \|\mathbf{A}\bar{T} - \bar{X}\|$$

$$\text{subject to } \mathbf{C}\bar{T} = \bar{b}.$$

- Tests with variable number of slices/hue regions/partitions show a delta E minimum for certain number of partitions

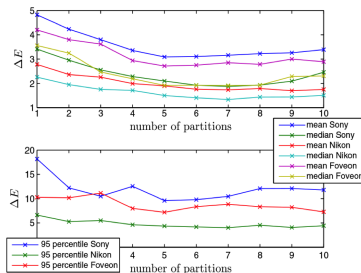


Fig. 8. Mean, median, and 95 percentile ΔE errors for the increasing number of hue partitions for the SG chart dataset.

NHPPCC - Results

- Sample results: Xrite SG Color Checker patches for training and evaluation, daylight standard illumination, reference measured with spectrophotometer PR-650; compared with other matrix based corrections and one Look Up Table (LUT) method:

Table 3. Nikon D70 and Sigma SD15 Characterization Results^a

Model Type	Nikon D70			Sigma SD15		
	Mean	Med	95 Pt.	Mean	Med	95 Pt.
LCC	2.5	2.3	5.1	5.2	4.0	16
PCC,2	2.1	1.8	4.8	3.9	3.2	9.4
PCC,3	1.7	1.6	3.1	3.1	2.3	7.0
LUT	1.7	1.4	4.4	4.3	3.3	12
RPCC,2	1.9	1.7	4.0	3.8	2.8	8.7
RPCC,3	1.6	1.3	3.4	3.2	2.4	9.1
HPPCC	2.5	2.1	6.0	5.0	3.2	17
NHPPCC-4	2.0	1.8	4.3	3.9	3.0	10
NHPPCC-6	1.9	1.7	3.9	3.5	2.6	9.1

^aThe error statistics are given as in Table 1.

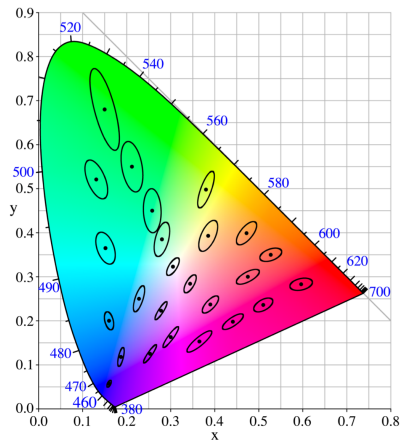
- Summary: results are close to methods with minimum delta E, but NHPPCC offers hue plane preserving and invariance to lightness scaling
 - Open question: is C0 continuity enough for smooth colour changes crossing the section borders

Perception Based Camera Characterization

Perception Based Camera Characterization I

Till now we optimized in tristimuli domain, but remember that the euclidian distance is not proportional to perception...

Recap: Colour Differences



- Chromaticities (left chromaticity diagram)

$$x = \frac{X}{X + Y + Z}, y = \frac{Y}{X + Y + Z}, z = \frac{Z}{X + Y + Z}$$

- Mac Adams 1942, experiment for just noticeable difference (JND), Mac Adams Ellipsis (left figure)
- CIE XYZ is not perceptual equidistant, same perceptual colour differences have different Euclidian distances in CIE XYZ domain

https://de.wikipedia.org/wiki/MacAdam-Ellipse#/media/File:CIExy1931_MacAdam.png

Recap: Perceptual Domain and Colour Difference

- Colour Appearance Model (CAM), perceptually uniform colour space

- Quantitative description in perceptual domain
split into lightness and chrominance:

CIE 1976 $L^*a^*b^*$ (CIE-Lab)

- $(X_n \ Y_n \ Z_n)^T$ as „White Point“
- Mac Adams Ellipsis now nearly circles of same diameter

$$L^* = 116f_{Lab}(Y/Y_n) - 16$$

$$\begin{aligned} a^* &= 500 (f_{Lab}(X/X_n) - f_{Lab}(Y/Y_n)) \\ b^* &= 200 (f_{Lab}(Y/Y_n) - f_{Lab}(Z/Z_n)). \end{aligned}$$

$$f_{Lab}(x) = \begin{cases} x^{\frac{1}{3}}, & \text{if } x > (\frac{6}{29})^3 \\ x^{\frac{841}{108}} + \frac{4}{29}, & \text{if } x \leq (\frac{6}{29})^3 \end{cases}$$

- Distance metric: Delta E 1976 colour difference $\Delta E = \sqrt{(L_1^* - L_2^*)^2 + (a_1^* - a_2^*)^2 + (b_1^* - b_2^*)^2}$
- Further optimized in CIE Delta E 2000 formulae

Perceptual Based Camera Characterization

- We are looking for the linear 3x3 matrix B by a solution for following *nonlinear* problem as basically proposed in the Academy Color Encoding System (ACES) and further modified here:

$$S = \sum_{i=1}^n ||f_{CAM}(x'_i, w_{ACES}) - f_{CAM}(MBv_i, w_{ACES})||$$

S Sum of errors, to be minimized e.g. through Levenberg-Marquardt algorithm

n Number of spectral reflectance of objects

f_{CAM} Function converting tristimulus into an **Colour Appearance Model** (e.g. CIE-Lab, CIE-CAM02)

x'_i Tristimulus of i^{th} object (defined by SPD of standard illuminant, spectral reflectance of object, •spectral response of standard observer; adapted chromatically to ACES white point)

w_{ACES} CIE-XYZ tristimulus of ACES white point

M 3x3 transformation matrix from ACES into CIE-XYZ domain (defined by ACES standard)

v_i Camera value of i^{th} object (derived by **SPD of real scene illumination (modification of previously published method or SPD from developed database**, spectral reflectance of object, **spectral response of camera (estimated with developed Low Budget hardware)**; target white point normalized

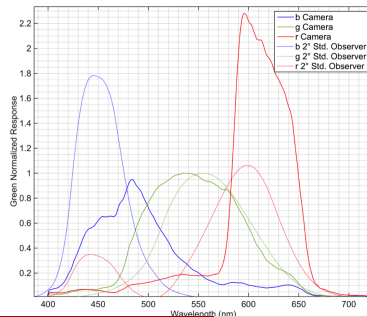
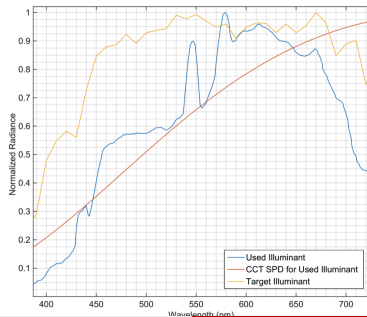
Minimizing the Perception Error

- This represents a non linear minimization problem
- Cannot be solved by using matrix inversion or, respectively, normal equation
- Commonly used algorithm for numeric solution:
LEVENBERG-MARQUARDT

OFTex Modification of Optimization

- optimization, whereas the scene illumination is taken into account for optimization
- Sample of mixed scene illumination (left bottom – Used Illuminant) and estimated camera response (right bottom – Camera)

$$\vec{C}_{Cam} = \begin{pmatrix} R_{Cam} \\ G_{Cam} \\ B_{Cam} \end{pmatrix} \sim \begin{pmatrix} \sum_{i=1}^n \phi_{Scene}(\lambda_i) \rho(\lambda_i) S_R(\lambda_i) \\ \sum_{i=1}^n \phi_{Scene}(\lambda_i) \rho(\lambda_i) S_G(\lambda_i) \\ \sum_{i=1}^n \phi_{Scene}(\lambda_i) \rho(\lambda_i) S_B(\lambda_i) \end{pmatrix}$$



Viewing a Profile Sample

- Lets take a look into a profile talk about what a "Gamut" represents...

Chart Sample of Profile Application I

Raw vs. Profiled

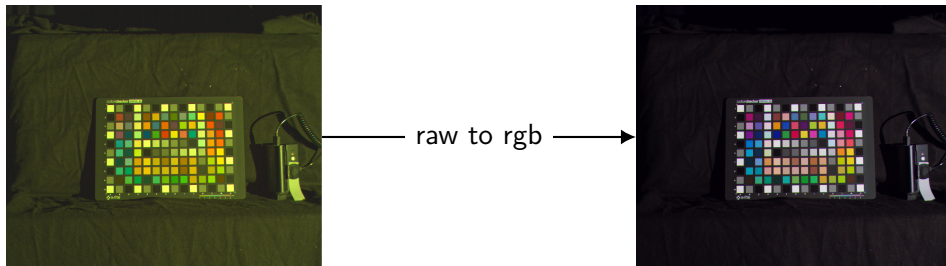
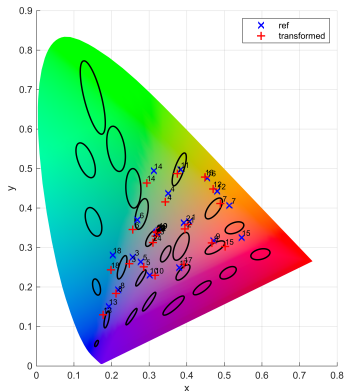
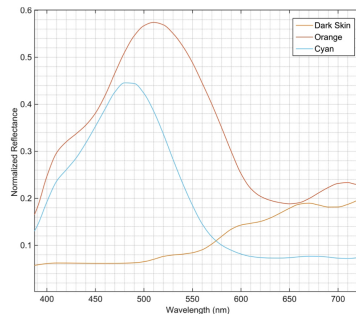


Chart Sample of Profile Application II

Reference vs. Profiled Values of Color Checker Patches in Chromaticity Domain



OFTex: Better Matching for Scene Illumination



- GoPro Hero Sample of a scene and Color Checker test chart (top left)
- Color Checker patches as training spectra
- Comparing colour corrected images, using the real scene illumination (spec) and correlated colour temperature (CCT) based equivalent radiation (CCT SPD for used Illuminant)
- Improvement examples: three Color Checker patches (top right CIE-Lab values, the ΔE_{2000} to the CIE standard observer and $\Delta(\Delta E_{2000})$ improvement (table below)

	L_{CCT}	a_{CCT}	b_{CCT}	L_{spec}	a_{spec}	b_{spec}	$\Delta E_{2000,CCT}$	$\Delta(\Delta E_{2000})_{spec-CCT}$
Dark Skin	44.65	10.74	24.44	44.58	11.31	23.49	9.21	-0.70
Orange	67.88	30.75	68.00	67.72	32.27	63.25	6.76	-1.77
Cyan	48.50	-16.68	-23.55	48.69	-17.94	-22.41	6.18	-0.60

Results of Open Film Tools Application in Karge, Open Film Tools - a Free Toolset for a Spectral Data Based Movie Camera Colour Characterization: 22. Workshop Farbbildverarbeitung, Ilmenau, 2016

Movie Sample of Profile Application

Movie



Colour management which uses spectral data based camera characterisation.

Summary

V6 Colour Reconstruction

Summary

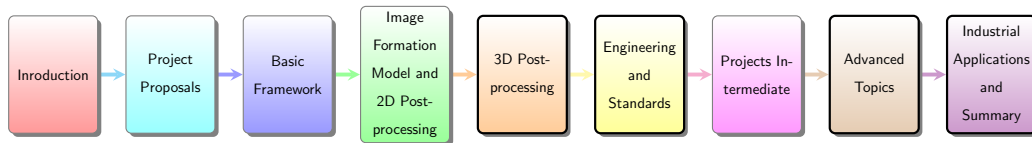
- The difference between human and camera observer
- Method for camera sensitivity estimation
- Methods for camera characterization

Take Aways

Camera characterisation is the fundamental method for color reconstruction, i.e. colour management.

Day 8

V7 2D Image Postprocessing



Content I

- 8 V7 2D Image Postprocessing
 - Visual Improvement Methods
 - Feature Detection
 - Image Segmentation
 - Coordinate Transformations

Since this semester the focus was on image formation and reconstruction here (only) a rough overview of selected image postprocessing methods is presented for:

- Visual improvement
- Feature detection and segmentation

Afterwards geometric transformations will be discussed.

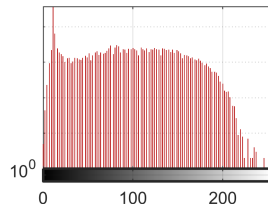
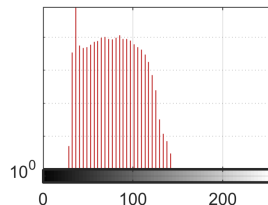
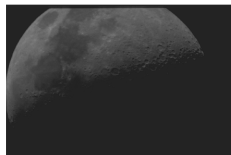
Visual Improvement Methods

Intensity Scaling

- Modification of pixel value as a function of old pixel value and optional constraints
- Function: Any function preserving maximum range, realized as discrete Look Up Table (LUT)
- Constraints: Minimum/Maximum of old values, probability (histogram)
- Artefacts: Clipping, quantization

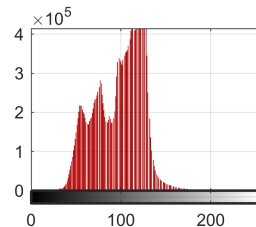
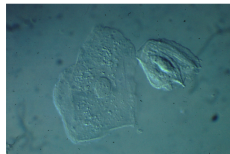
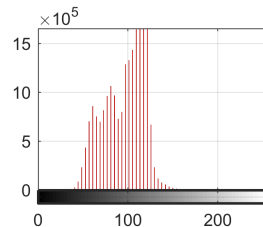
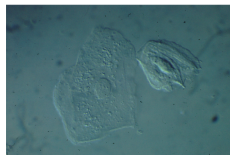
Histogram Spreading

- Spread values to output range
- Correction of wrong exposure
- Adapt to different ranges



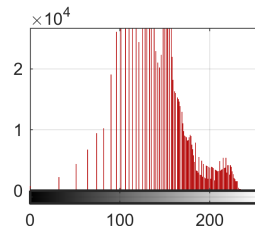
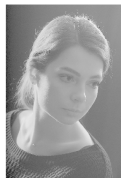
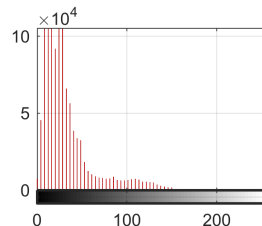
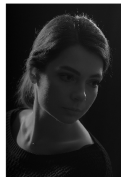
Exponential Scaling

- Exponential or potency function
- Increase contrast



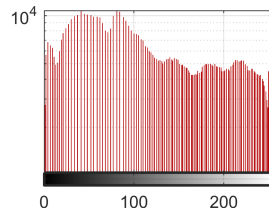
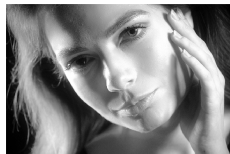
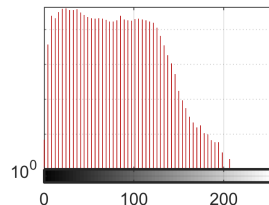
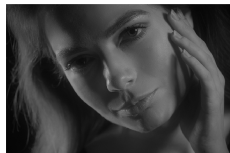
Logarithmic Scaling

- Log function, be aware of minimum value!
- Better visualisation of shadow areas



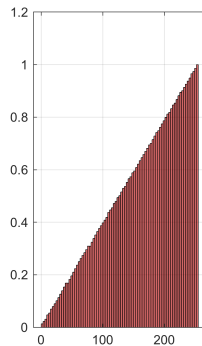
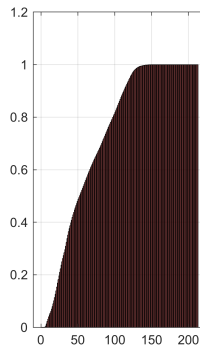
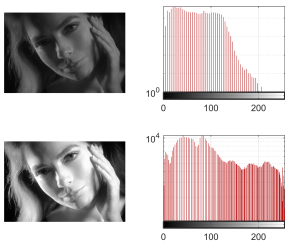
Histogram Equalization

- Function to use all available output values in evenly distributed manner
- Improve visualisation of distinguished areas



Histogram Equalization II

The cumulative histogram will be linearised (right view: old vs. equalized).



Filter

- Linear filter: convolution in spatial domain/multiplication in frequency domain
- Ranking filter, bilateral filter: not representable by multiplication in frequency domain

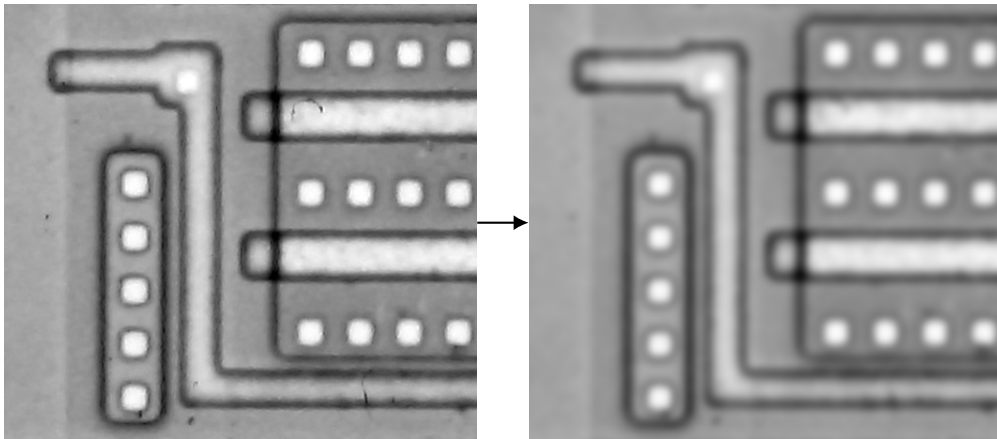
Linear Filter for Smoothing - GAUSS

Suppresses higher frequencies by kernel function: $h = \frac{1}{2\pi\sigma^2} \exp -\frac{x^2+y^2}{2\sigma^2}$

Discrete convolution kernel example:

$$h = \frac{1}{16} \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \frac{1}{4} \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$$

GAUSS-Filter Sample



Linear Filter for Smoothing

- Common aspect: Suppressing higher frequencies
- Other linear filters: Boxing filter, sinc filter

Linear Filter

Artefacts:

- Sampling errors
- Anisotropic behaviour, discrete filter kernel not rotation invariant
- Ringing on edges

Ranking Filter

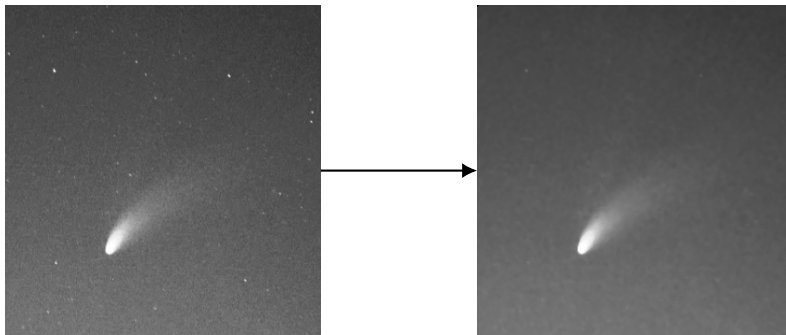
Class of non linear filters

- 1 Collect all pixel values in defined neighbourhood in a list
- 2 Order this list applying a sort criteria
- 3 Select a list member as new pixel value

Sorting the list in ascending order following filters can be obtained:

Ranking Filter: Median

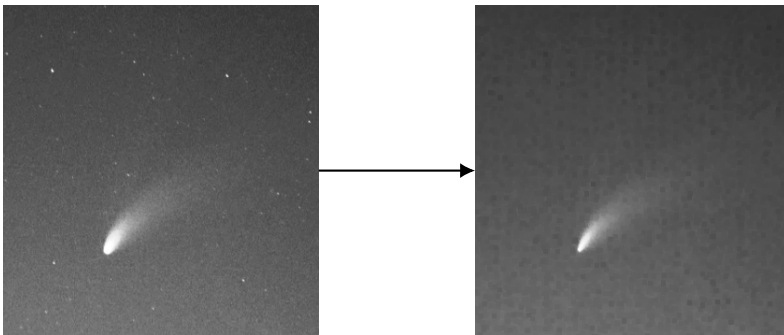
Medianfilter, select value in the middle of the list



Smoothing with robustness against outliers

Ranking Filter: Minimum

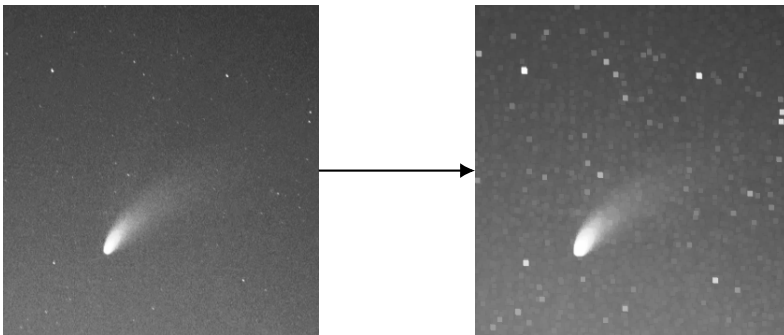
Minimumfilter, select minimum value in list



Closing of areas, also called Erode-Filter

Ranking Filter: Max

Maximumfilter, select maximum value in list



Opening of areas, also called Dilate-Filter

Denoising and Edge Preserving Methods

Visual improvement often requires not only noise reduction but also edge preserving. Now we take a look at one non linear filter and one algorithm, which are commonly used for that purpose.

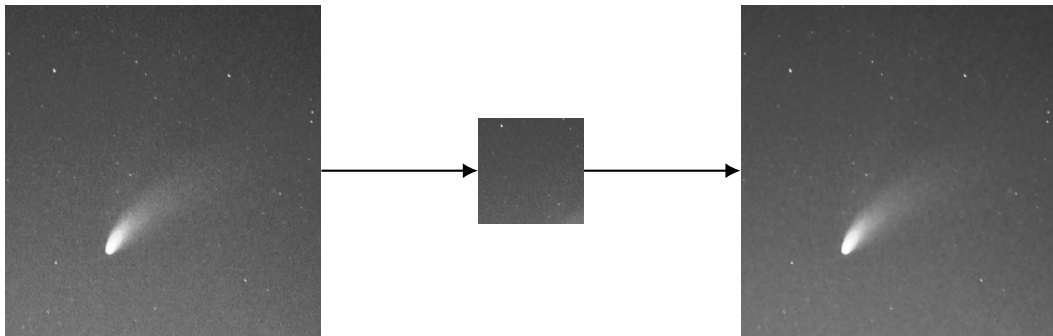
Bilateral Filter

Another class of nonlinear filter

- Combination of GAUSS kernels weighted for geometrical and lightness distance
- Meaningful GAUSS kernel can be estimated by noisy region

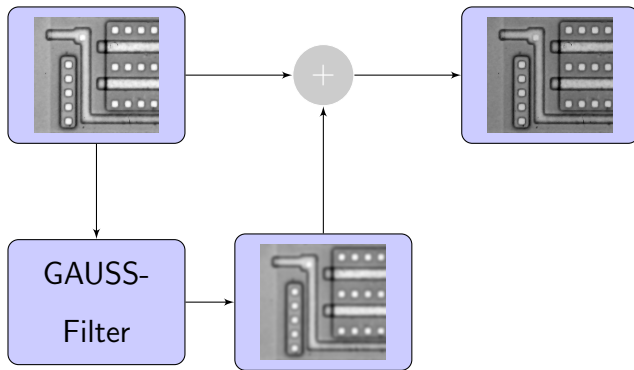
Bilateral Filter Sample

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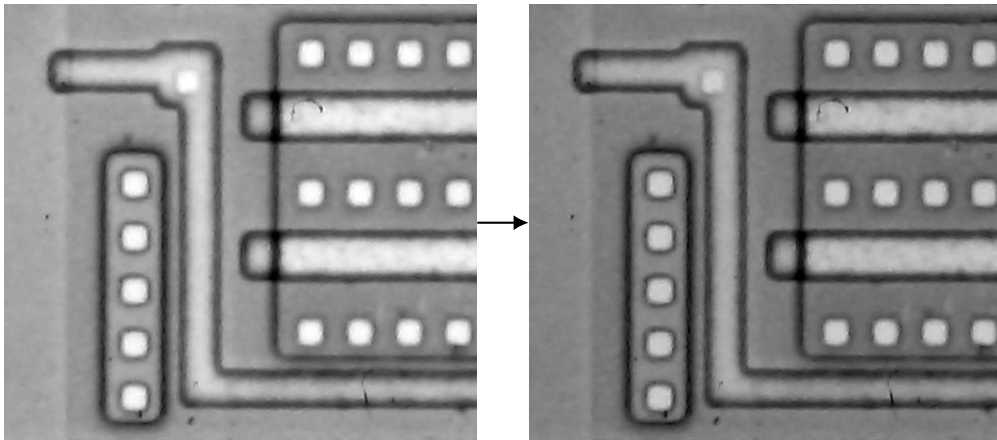


Unsharp Masking Algorithm

- A combination of initial image with its blurred version
- Degrees of freedom: weighting the added unsharp image and GAUSS-Kernel



Unsharp Masking Sample



Feature Detection

Feature Detection

Image Features

Edges

Corners

Blobs

Ridges
(Lines)

Feature Detection

Pixel properties, beside its intensity:

- first derivate, its magnitude and direction, structure tensor
- second derivative
- local contrast
- local frequency spectrum
- ...

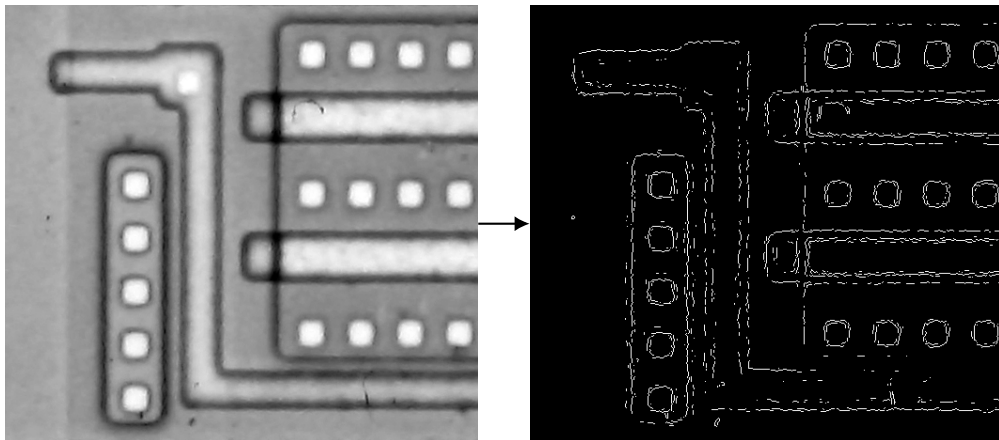
They are used to assign a pixel to a certain feature.

Linear Filter for Edge Detection - SOBEL

First derivative with convolution kernels: horizontal (h), vertical (v)

$$h_h = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}, h_v = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

SOBEL-Filter Sample



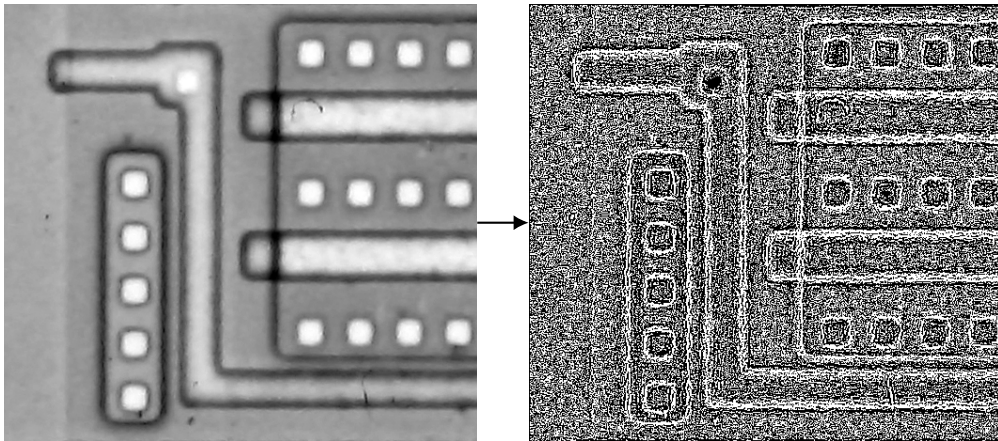
Linear Filter for Edge Detection - LAPLACE

Second derivative with convolution Kernels: h

$$h = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}, h_d = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

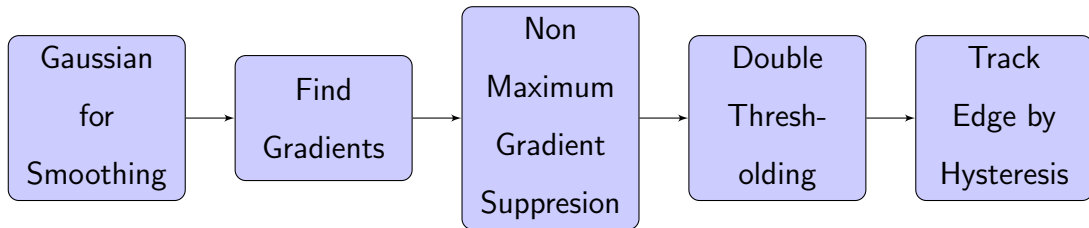
h_d diagonal awareness

LAPLACE-Filter Sample



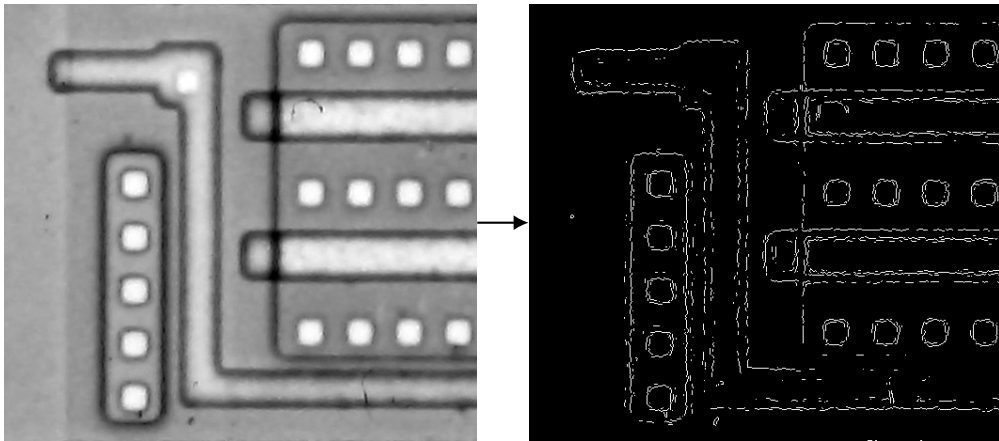
Edge Detectors - CANNY

Algorithm:



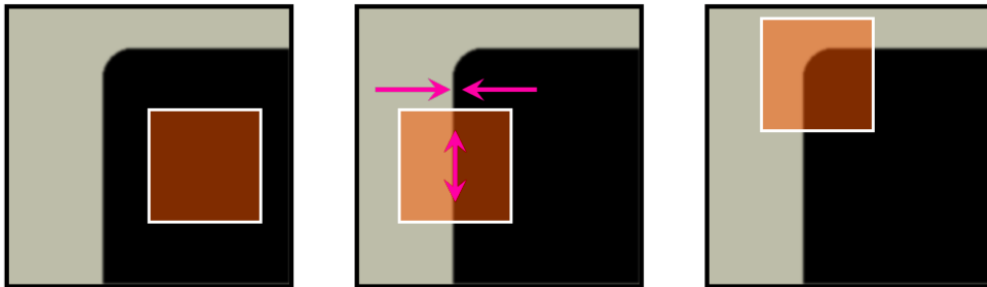
Advantage: smooth edge detection

Edge Detectors - CANNY



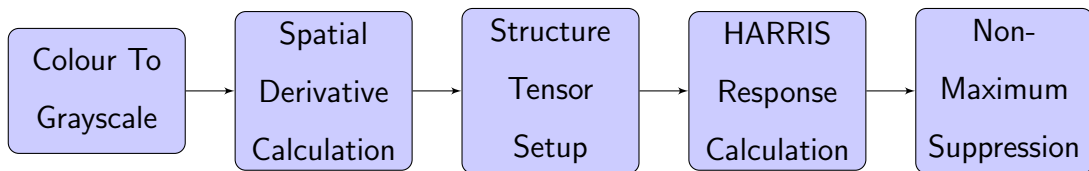
Corner Detection

What is a corner?



Corner Detection - HARRIS-Corner Detector

Algorithm:



HARRIS-Corner Detector: Structure Tensor

Based on SSD and using the derivatives I_x , I_y and a window region W around a pixel, the structure tensor is:

$$\text{Rotate: } T = \sum_{(x,y \in W)} \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix} = \begin{pmatrix} \sum_{(x,y \in W)} I_x^2 & \sum_{(x,y \in W)} I_x I_y \\ \sum_{(x,y \in W)} I_x I_y & \sum_{(x,y \in W)} I_y^2 \end{pmatrix}$$

HARRIS-Corner Detector: HARRIS Response

Then, the HARRIS response R can be calculated:

$$R = \lambda_{min} \approx \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{\det(T)}{\text{tr}(T)}$$

HARRIS-Corner Detector Sample

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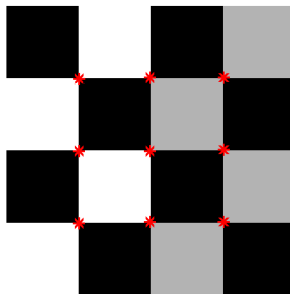


Image Segmentation

Image Segmentation

Segmentation Methods

Thresholding

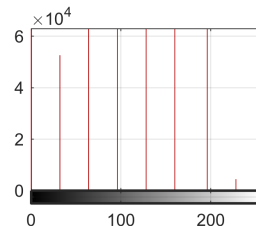
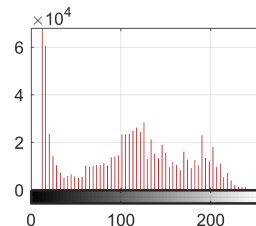
Quantization

n-modal
histogram
based

...

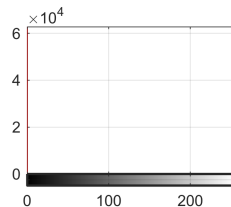
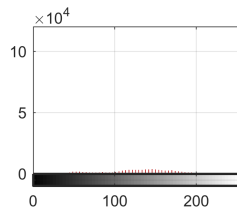
Histogram Quantization

- Decrease the used values
- Area visualisation, object detection



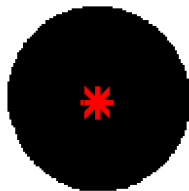
Histogram Binarization

- Special case of quantisation
- Threshold can be derived from histogram information



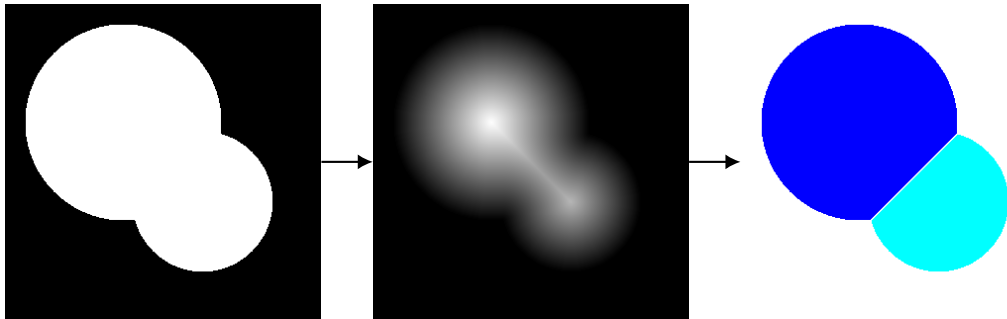
Center of Gravity

Center coordinate of segment: Mean value of all horizontal/vertical centers.



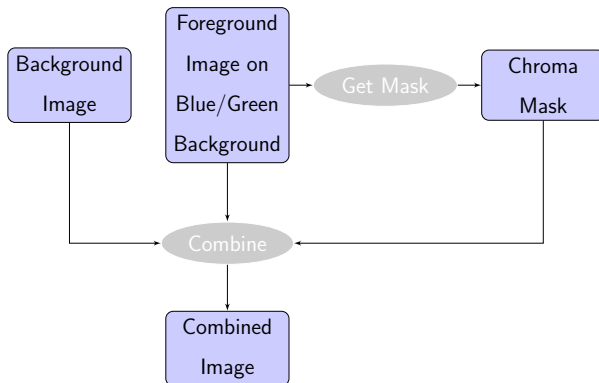
Watershed

...but what about overlapping? Pour out water on top of a mountain...



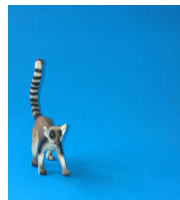
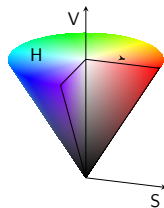
Application Sample: Chroma Keying

An algorithm for image combining using color as segmentation criteria:



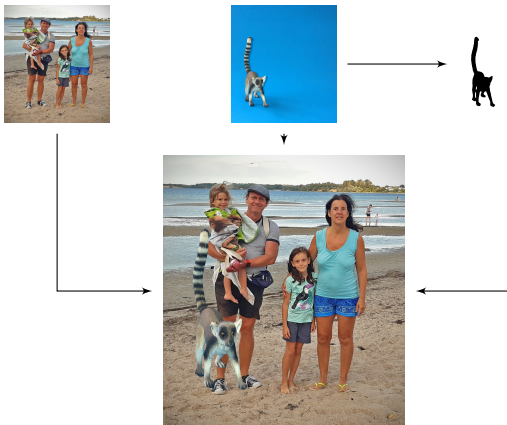
Chroma Selection Criteria

- RGB-Image must be converted in HSV-Color Domain.
- HSV Domain: Hue, Value and Saturation Separation
- The Hue (chroma) of background is used for separation.



Sample

Chroma Keying Sample



Coordinate Transformations

Coordinate Transformations

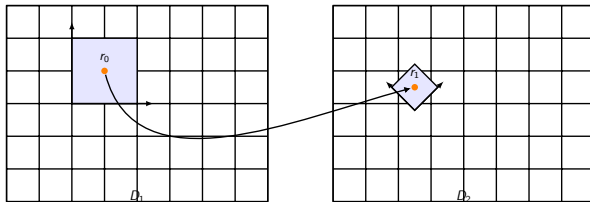
- Affine Transformation
- Projective Transformation
- Polar/Cartesian Transformation

Affine Transformation

$\mathbf{r}_0 = (u_0, v_0)^T$ image coordinates, $\mathbf{r}_1 = (u_1, v_1)^T$ transformed image coordinates

$$\mathbf{A} : \mathbf{r}_0 \mapsto \mathbf{r}_1$$

Affine transformation $\mathbf{A}$: three collinear points are themselves collinear



D_2

Affine Transformation

Preserved:

- Parallelism
- Collinearity: awareness of straight lines
- Incidence: a point still lies on a curve
- Ratios along a line, especially midpoints

Affine Transformation

Unpreserved:

- Lengths
- Angles
- Areas

Affine Transformation

$$\textit{Identity:} \quad \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$$

$$\textit{Scale:} \quad \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} s_u & 0 \\ 0 & s_v \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}, \textit{ scaling factors: } s_u, s_v$$

Affine Transformation

$$\textit{Shift:} \quad \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \begin{pmatrix} t_u \\ t_v \end{pmatrix}$$

$$\textit{Rotate:} \quad \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$$

Affine Transformation

Shear

$$\begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} 1 & c_v \\ c_u & 1 \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$$

Affine Transformation

Combined, for camera with isotrope scaling and no shear as an approximation

$$\begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} scos(\phi) & s - sin(\phi) \\ ssin(\phi) & scos(\phi) \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \begin{pmatrix} t_u \\ t_v \end{pmatrix}$$

Note: order is important

Affine Transformation

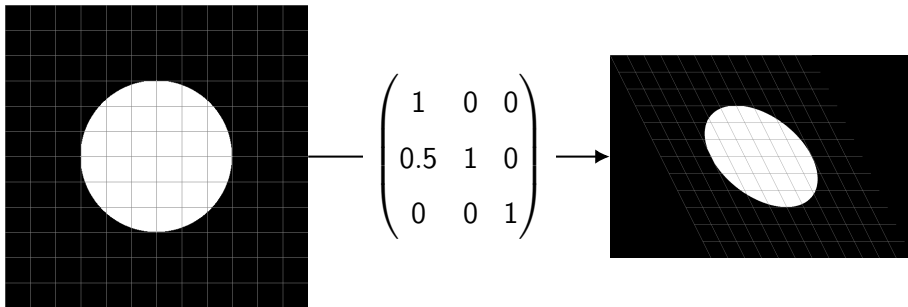
Combined in Homogeneous Coordinates

$$\begin{pmatrix} u_1 \\ v_1 \\ w_1 \end{pmatrix} = \begin{pmatrix} s_u \cos(\phi) & c_v - \sin(\phi) & t_u \\ c_u \sin(\phi) & s_v \cos(\phi) & t_v \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \\ w_0 \end{pmatrix}$$

Recap: this form has advantage to have a matrix only based model of transform

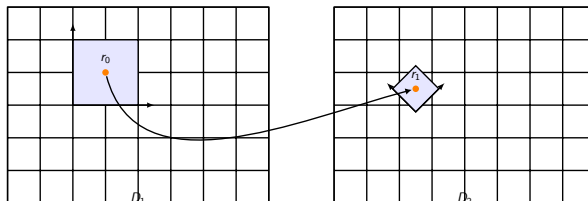
Affine Transformation

Apply Transform: Sample



Affine Transformation

Sampling/Inverse Sampling Problem for Pixel Value Reconstruction

 D_2

Solution: Interpolation methods as discussed in demosaicing

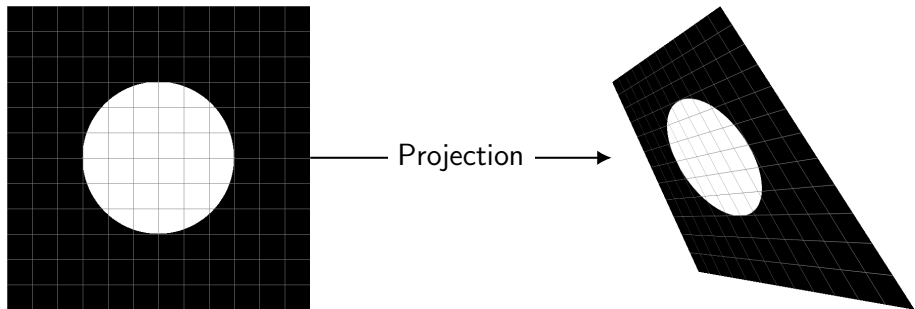
Projective Transformation

Superset of affine transform, general form:

$$\begin{pmatrix} u_1 \\ v_1 \\ w_1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \\ w_0 \end{pmatrix}$$

Note: Especially last row elements can be different to zero

Projective Transformation Sample



Projective Transformation

Two applications:

- Apply a transform
- Estimate transformation matrix and apply inverse transformation

Essential preprocessing for last item: Find correlated points by manual setting or automatic registration with feature detection and correlation methods.

Projective Transformation

Cartesic coordinates are given as:

$$u_1 = \frac{au_0 + bv_0 + c}{gu_0 + hv_0 + i}, v_1 = \frac{du_0 + ev_0 + f}{gu_0 + hv_0 + i}$$

Approximation with $i = 1$:

$$u_1 = \frac{au_0 + bv_0 + c}{gu_0 + hv_0 + 1}, v_1 = \frac{du_0 + ev_0 + f}{gu_0 + hv_0 + 1}$$

Projective Transformation

Note, if $g = 0, h = 0, c = t_u, f = t_v$, it gives affine transformation:

$$u_1 = \frac{au_0 + bv_0 + c}{gu_0 + hv_0 + 1},$$
$$v_1 = \frac{du_0 + ev_0 + f}{gu_0 + hv_0 + 1}$$

Projective Transformation

Normalize every coefficient with coefficient i ($\tilde{z} = \frac{z}{i}$, $z = a..i$) gives:

$$u_1 = \frac{\tilde{a}u_0 + \tilde{b}v_0 + \tilde{c}}{\tilde{g}u_0 + \tilde{h}v_0 + 1}, v_1 = \frac{\tilde{d}u_0 + \tilde{e}v_0 + \tilde{f}}{\tilde{g}u_0 + \tilde{h}v_0 + 1}$$

Now one can write:

$$u_1 = \tilde{a}u_0 + \tilde{b}v_0 + \tilde{c} - \tilde{g}u_1u_0 - \tilde{h}u_1v_0$$

$$v_1 = \tilde{d}u_0 + \tilde{e}v_0 + \tilde{f} - \tilde{g}v_1u_0 - \tilde{h}v_1v_0$$

Projective Transformation

...and can calibrate/estimate matrix with a set of points using:

$$\begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} u_0 & v_0 & 1 & 0 & 0 & 0 & -u_1 u_0 & -u_1 v_0 \\ 0 & 0 & 0 & u_0 & v_0 & 1 & v_1 u_0 & v_1 v_0 \end{pmatrix} \begin{pmatrix} \tilde{a} \\ \tilde{b} \\ \tilde{c} \\ \tilde{d} \\ \tilde{e} \\ \tilde{f} \\ \tilde{g} \\ \tilde{h} \end{pmatrix}$$

Projective Transformation Application

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Rectification, e.g. for architecture



Rectify →



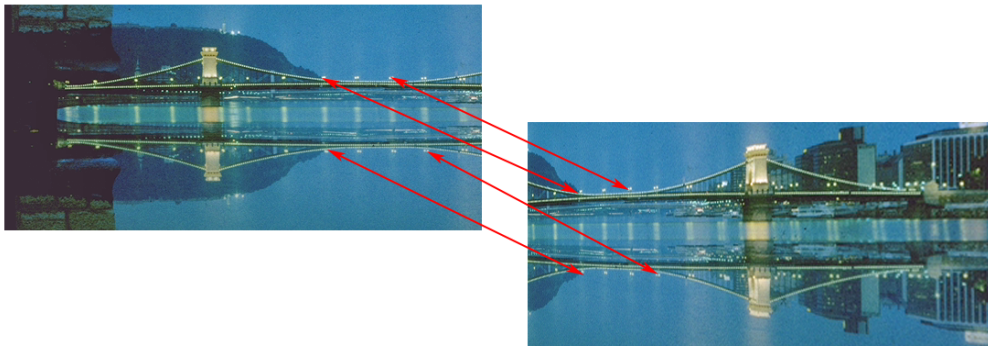
Projective Transformation Application

Stitching: Sequence of images



Projective Transformation Application

Stitching: Matching points



Projective Transformation Application

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Stitching: Result



Polar/Cartesian Transformation

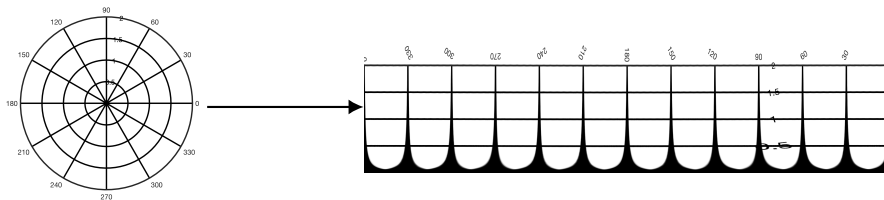
Not all images are Cartesian! Polar to Cartesian transformation is given by:

$$\begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = r \begin{pmatrix} \cos(\phi) \\ \sin(\phi) \end{pmatrix}, \text{ with } r = \sqrt{u_0^2 + v_0^2}, \phi = \tan(v_0/u_0)$$

Applications: Fisheye/Sphere image transform

Polar/Cartesian Transformation

Abstract Sample

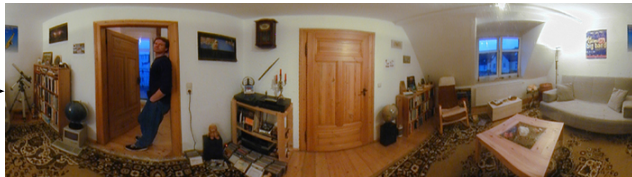


Preprocessing: Find center and horizon; **Postprocessing:** Distortion correction for inclination (lower/upper viewing angle); **Additional problem:** Different resolutions

Polar/Cartesian Transformation Sample

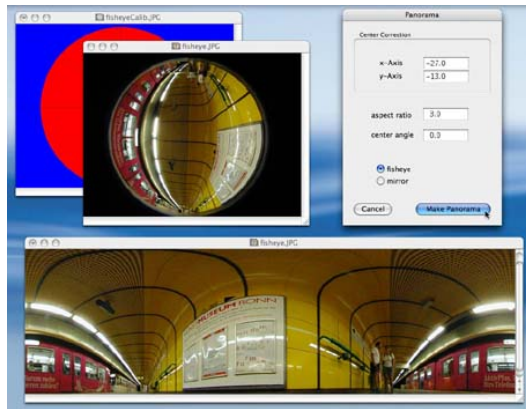
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Real World Scenario



Polar/Cartesian Transformation Fishe-Eye Sample

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Pre/Postprocessing: Find center; Distortion correction for inclinations

Summary

V7 2D Image Postprocessing

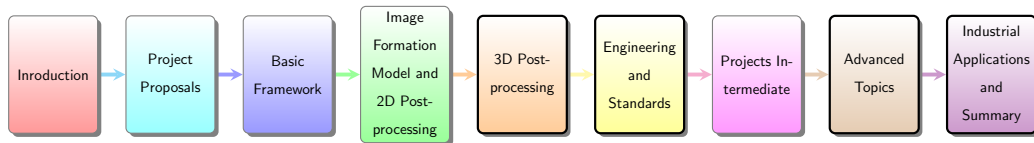
Summary

- Improvement of visual appearance
- Feature detection and segmentation
- Geometrical transforms

Take Aways

Postprocessing should be done always in a linear domain, even if the result is evaluated by human eye.

V8 3D Object Reconstruction, Displaying, Engineering and Summary



Content I

- 9 V8 3D Object Reconstruction, Displaying, Engineering and Summary
 - 3D Object Reconstruction
 - Standardized Output Display Colour Management
 - Engineering and Standards
 - Advanced Topic: Polarization-RGB Cameras
 - Industrial Applications Examples

Focus

- 3D Object Reconstruction
- Output Display Colour Calibration
- A State of the Art Topic: Polarization RGB Sensors
- Engineering, Standards, Applications and Final Summary

3D Object Reconstruction

3D Object Reconstruction for Observers

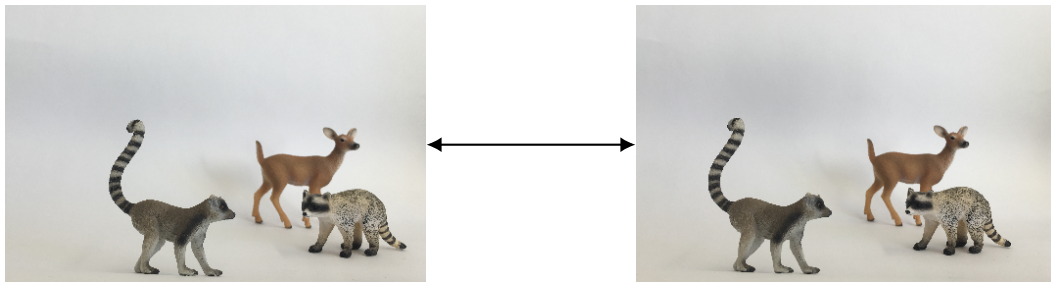
Beside the relative horizontal and vertical position, human observers also realize a relative depth estimation. How the human observer estimates relative depth of objects:

- Objects shadings
- Perspective lines, ratios
- Motion

This can be done with one eye only, but ...

Stereo Vision

... the human observer estimates relative depth of objects also by stereo vision.



Stereo Vision



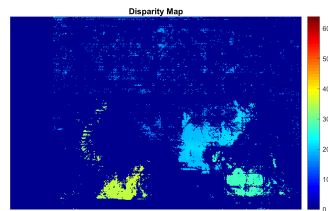
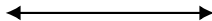
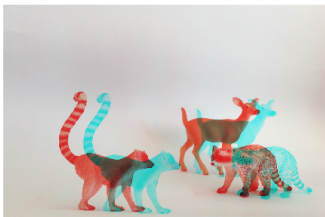
A parallax can be observed. It forms a disparity map in visual cortex.

Main Tasks in Stereo Vision

- Finding Disparities
- Solving Correspondence Problem

Stereo Vision

Disparity map showing relative distances



Stereo Image Pairs Presentation

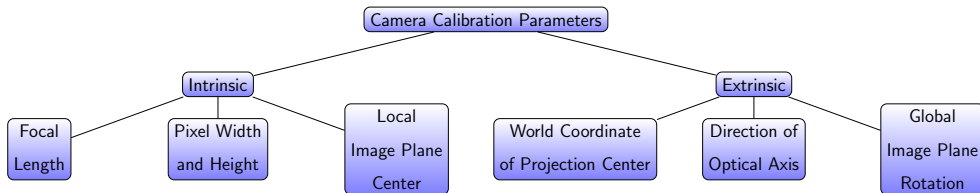
Application of stereo image pairs in photography/movie production

- Side by side presentation in visual display systems
- Methods: anaglyph, polarisation

But in machine vision the real world coordinates must be obtained.

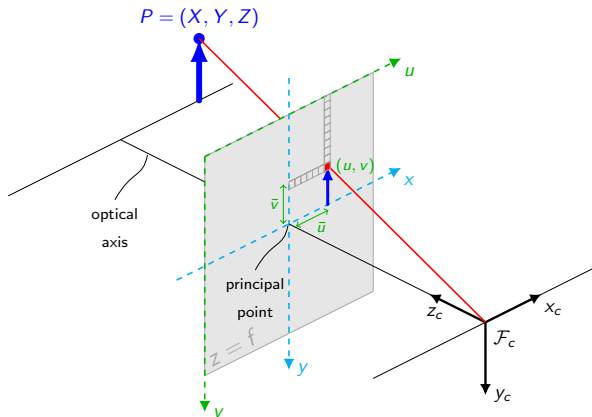
From Camera Coordinates to World Coordinates

Previously only 2D images object relations were discussed. Now we like to obtain absolute 3D coordinates of objects. For that the camera must be calibrated.



Recap: 3D Pin Hole Camera Geometry

Image plane is swapped:



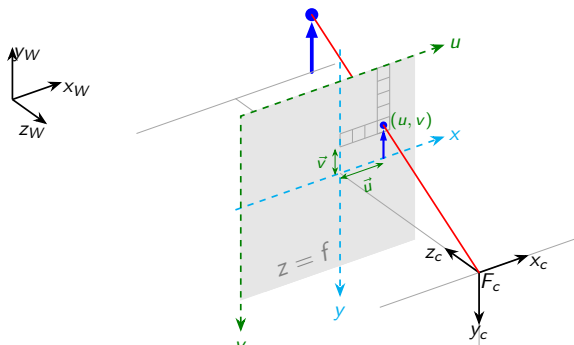
Intrinsic Parameters

Camera properties: pixel indices, pixel width and height, index offset to center position, focal length. The relation from pixel index to real world coordinates $(X, Y, Z)^T$ is then

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \frac{f}{u_e} & 0 & t_u \\ 0 & \frac{f}{v_e} & t_v \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{X}{Z} \\ \frac{Y}{Z} \\ 1 \end{pmatrix} = \mathbf{P}_c \frac{Z}{Z} \begin{pmatrix} \frac{X}{Z} \\ \frac{Y}{Z} \\ 1 \end{pmatrix} = \mathbf{P}_c \frac{1}{Z} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Extrinsic Parameters

$$P = (X_W, Y_W, Z_W)$$



But if the real world coordinates are defined somewhere else, position and orientation, i.e. the camera pose must be also determined.

Extrinsic Parameters

Camera pose properties: 3D projection center translation, direction and rotation defined by:

$$\mathbf{W} = \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & t_X \\ r_{2,1} & r_{2,2} & r_{2,3} & t_Y \\ r_{3,1} & r_{3,2} & r_{3,3} & t_Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Camera Calibration

The whole transformation written in homogeneous coordinates

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \mathbf{P}_C \mathbf{W} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{f}{u_e} & 0 & t_u & 0 \\ 0 & \frac{f}{v_e} & t_v & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & t_x \\ r_{2,1} & r_{2,2} & r_{2,3} & t_y \\ r_{3,1} & r_{3,2} & r_{3,3} & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Camera Calibration

Therefore the coefficients of $\mathbf{P}_c \mathbf{W}$ are (setting $\phi_u = f/u_e, \phi_v = f/v_e$):

$$\mathbf{P}_c \mathbf{W} = \begin{pmatrix} \phi_u r_{1,1} + t_u \phi_u r_{3,1} & \phi_u r_{1,2} + t_u \phi_u r_{3,2} & \phi_u r_{1,3} + t_u \phi_u r_{3,3} & \phi_u t_X + t_u t_Z \\ \phi_v r_{2,1} + t_v \phi_u r_{3,1} & \phi_v r_{2,2} + t_v \phi_u r_{3,2} & \phi_v r_{2,3} + t_v \phi_u r_{3,3} & \phi_v t_Y + t_v t_Z \\ & r_{3,1} & r_{3,2} & r_{3,3} & t_Z \end{pmatrix}$$

$$= \begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} \end{pmatrix} = \mathbf{M}$$

Camera Calibration Target

How we can estimate the calibration matrix?

- Using a calibration target with known coordinates
- Solving equation by direct linear transform



Find Matrix by Direct Linear Transform

Coordinates for a point i are given as:

$$u_i = \frac{m_{1,1}X_i + m_{1,2}Y_i + m_{1,3}Z_i + m_{1,4}}{m_{3,1}X_i + m_{3,2}Y_i + m_{3,3}Z_i + m_{3,4}}, v_i = \frac{m_{2,1}X_i + m_{2,2}Y_i + m_{2,3}Z_i + m_{2,4}}{m_{3,1}X_i + m_{3,2}Y_i + m_{3,3}Z_i + m_{3,4}}$$

$$\Rightarrow u_i(m_{3,1}X_i + m_{3,2}Y_i + m_{3,3}Z_i + m_{3,4}) = m_{1,1}X_i + m_{1,2}Y_i + m_{1,3}Z_i + m_{1,4}$$

$$\Rightarrow v_i(m_{3,1}X_i + m_{3,2}Y_i + m_{3,3}Z_i + m_{3,4}) = m_{2,1}X_i + m_{2,2}Y_i + m_{2,3}Z_i + m_{2,4}$$

Find Matrix by Direct Linear Transform II

...and one can calibrate/estimate matrix with a set of points using:

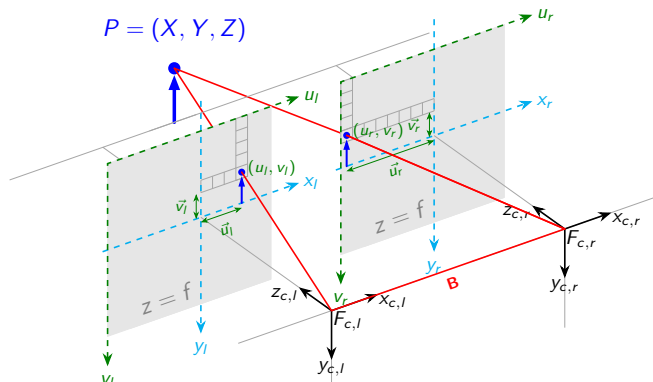
$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_i X_i & -u_i Y_i & -u_i Z_i & -u_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_i X_i & -v_i Y_i & -v_i Z_i & -v_i \end{pmatrix} \begin{pmatrix} m_{1,1} \\ m_{1,2} \\ m_{1,3} \\ m_{1,4} \\ m_{2,1} \\ m_{2,2} \\ m_{2,3} \\ m_{2,4} \\ m_{3,1} \\ m_{3,2} \\ m_{3,3} \\ m_{3,4} \end{pmatrix}$$

Camera Calibration Result

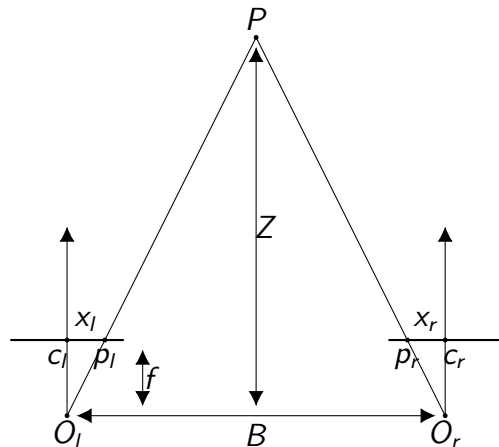
- Now we have the calibration matrix for 3D/2D relation.
- If the camera spans the world coordinates, one can directly derive the intrinsic parameter matrix $\mathbf{P}_C$.
- Note: $\mathbf{P}_C$ also might include the former discussed geometric distortions caused by lenses.

Stereo Vision by Technical Observer

Now a calibrated stereo camera system gives 3D coordinates



Depth from Stereo Image Pairs



With the stereo base B and registered left/right pixel locations x_l, x_r the depth Z can be calculated:

$$\frac{|x_l - x_r|}{f} = \frac{\Delta x}{f} = \frac{B}{Z}$$

$$Z = \frac{B}{\Delta x} f$$

How to Find Correspondences

1) Feature detection per image:

- Simple features: e.g. HARRIS-Corner detector
- Complex features: e.g. SIFT, SURF

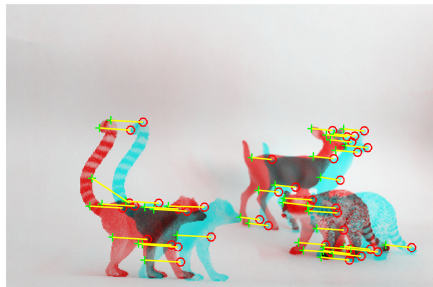
2) Find matches:

- (Normalized) Cross correlation
- Point vs. block based approaches

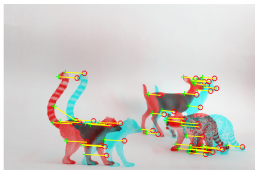
Sample of Found Correspondences

Right image shows HARRIS-Corner feature matches.

Note the *outlier* in the left ring-tailed cats tail!



How to Apply Found Correspondences



↓
3D Coordinates

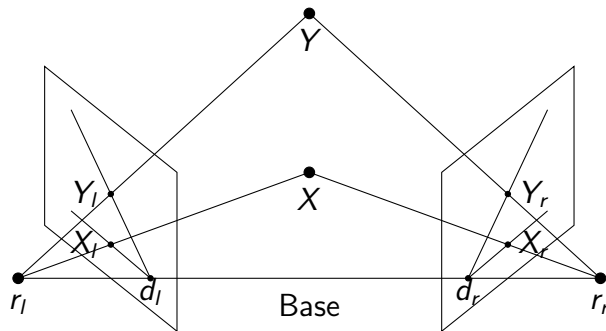
3) Estimate transformation matrix with matches:

- Least Square Solution
- Random sample consensus (RANSAC), an iterative method for estimating a mathematical model from a data set that contains *outliers*.

A More General Camera Arrangement

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Epipolar Geometry



Note: The second image can be captured by the same camera at different time!

Epipolar Geometry

This general camera arrangement requires following preprocessing steps before depth can be estimated from images:

- Lines with corresponding points must be found.
- Rectification: transform those lines to become horizontal (or vertical).

Epipolar geometry is commonly used in remote sensing, robotics, autonomous driving...

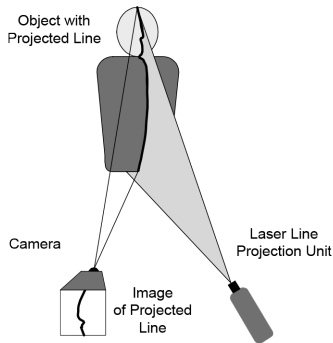
Active Triangulation using Scanning Illumination

Previous methods are called passive methods, now active methods are discussed, i.e. structured light illuminates the object:

- Derive structure from known illumination patterns, which are distorted by reflection on objects surface
- Patterns: Points, lines, grids with different spatial frequencies

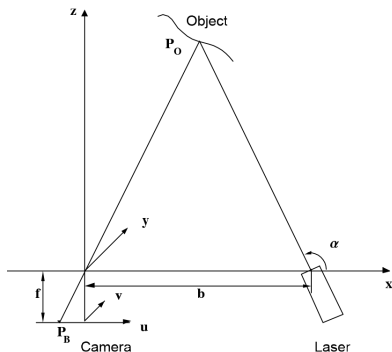
Line Projection Method

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But how the patterns can be detected? By a simple difference operation of two images with/without active illumination.

Line Projection Math



An object point P_O is the intersection of laser line plane with line of detected image pixel P_B through projection center.

Line Projection Math II

Two equations are defined as follows:

$$\text{Line: } g = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} x_B \\ y_B \\ -f \end{pmatrix}; \text{ Plane: } F = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} \cos\alpha \\ 0 \\ \sin\alpha \end{pmatrix} + \nu \begin{pmatrix} 0 \\ \cos\beta \\ \sin\beta \end{pmatrix}$$

Line Projection Math III

To be solved:

$$\begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & 0 & -x_B \\ 0 & \cos\beta & -y_B \\ \sin\alpha & \sin\beta & f \end{pmatrix} \begin{pmatrix} \mu \\ \nu \\ \kappa \end{pmatrix}$$

Line Projection Math IV

Object coordinates as solution:

$$\begin{pmatrix} x_O \\ y_O \\ z_O \end{pmatrix} = \frac{b \tan \alpha}{f + x_B \tan \alpha + y_B \tan \beta} \begin{pmatrix} x_B \\ y_B \\ f \end{pmatrix}$$

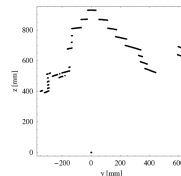
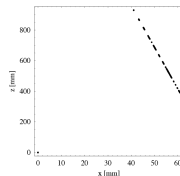
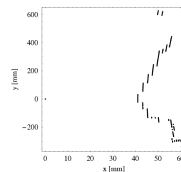
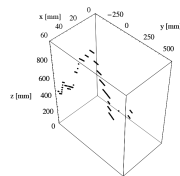
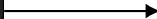
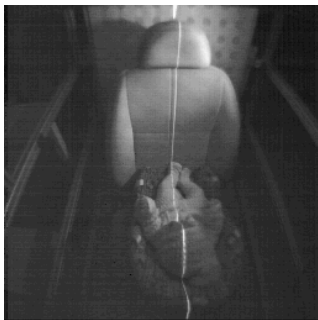
Line Projection Math V

Approximation

$$\begin{pmatrix} x_O \\ y_O \\ z_O \end{pmatrix} = \lim_{\alpha \rightarrow \frac{\pi}{2}, \beta \rightarrow 0} \frac{b \tan \alpha}{f + x_B \tan \alpha + y_B \tan \beta} \begin{pmatrix} x_B \\ y_B \\ f \end{pmatrix} = -\frac{b}{x_B} \begin{pmatrix} x_B \\ y_B \\ f \end{pmatrix}$$

Line Projection Sample: Occupant Sensing

Background: Air-bag trigger is switched off, if baby seat is detected.



Other 3D Reconstruction Methods

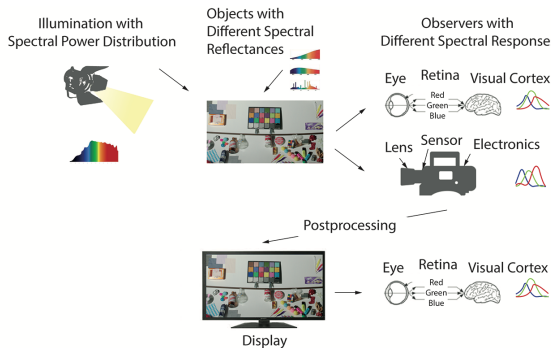
based on:

- Silhouette
- Shading
- Motion

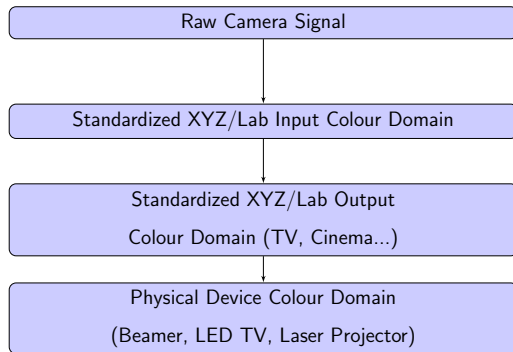
Standardized Output Display Colour Management

Image Presentation

At the end, the image is very often presented at a physical display device, where the display objects color might be the same as for the original objects.



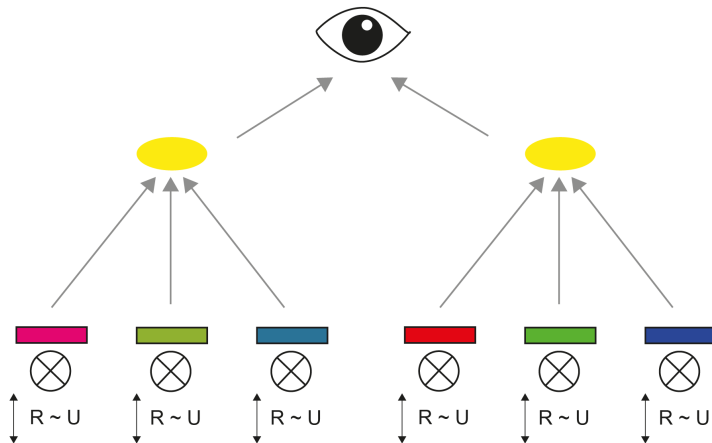
Whole Colour Image Transformation Pipeline



It is convenient to use an additional domain in between for standardized media deployment.

Recap: Colour Mixing Experiment

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Recap: Colour Transformation

A colour can be converted by a linear transformation:

$$\begin{pmatrix} R_1 \\ G_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} R_0 \\ G_0 \\ B_0 \end{pmatrix}$$

Colour Domain Transformation for Standards

- Photography: Adobe RGB
- Cinematography: ACES (Output Device)
- TV: REC 709, REC 2020

The transformation into this domains includes the linear transform, and very often an argument scaling for ranges, gamut mapping...

Colour Domain Transformation for Display

Calculation of required Tristimuli for Primaries

e.g. for Red

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix}_{LED} = \begin{pmatrix} \alpha x_R^{LED} & \beta x_G^{LED} & \gamma x_B^{LED} \\ \alpha y_R^{LED} & \beta y_G^{LED} & \gamma y_B^{LED} \\ \alpha z_R^{LED} & \beta z_G^{LED} & \gamma z_B^{LED} \end{pmatrix}^{-1} \begin{pmatrix} \kappa x_R^{TV} & \lambda x_G^{TV} & \mu x_B^{TV} \\ \kappa y_R^{TV} & \lambda y_G^{TV} & \mu y_B^{TV} \\ \kappa z_R^{TV} & \lambda z_G^{TV} & \mu z_B^{TV} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{TV}$$

Estimation of Coefficients

e.g. by GAUSS- Elimination

$$\begin{pmatrix} x_R & x_G & x_B \\ y_R & y_G & y_B \\ z_R & z_G & z_B \end{pmatrix} \begin{pmatrix} \rho \\ \sigma \\ \tau \end{pmatrix} = \begin{pmatrix} x_w/y_w \\ 1 \\ (1 - x_w - y_w)/y_w \end{pmatrix}$$

Colour Domain Transformation for Display II

Clipping, Scaling; Calculation of Primaries for White

Conversion of Colours into Radiances

$$\begin{pmatrix} \Phi_R \\ \Phi_G \\ \Phi_B \end{pmatrix} = \begin{pmatrix} \Phi_R^w & 0 & 0 \\ 0 & \Phi_G^w & 0 \\ 0 & 0 & \Phi_B^w \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}_{LED, clipped, scaled}$$

Linearisation for Display

The Electro Optical Transfer (EOT) function:

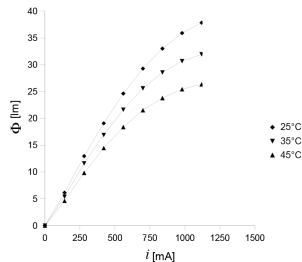
Estimation of current EOT

Polynomial of second order
estimated from measured EOTs

$$\begin{pmatrix} \check{\theta}^2 & \check{\theta} & 1 \\ \bar{\theta}^2 & \bar{\theta} & 1 \\ \hat{\theta}^2 & \hat{\theta} & 1 \end{pmatrix} \begin{pmatrix} a_2 \\ a_1 \\ a_0 \end{pmatrix} = \begin{pmatrix} \Phi(i, \check{\theta}) \\ \Phi(i, \bar{\theta}) \\ \Phi(i, \hat{\theta}) \end{pmatrix}$$

Calculation of required radiance

$$\Phi_d(i, \theta) = \frac{\Phi_{max}}{\Phi(i_{max}, \theta)} (a_2 \theta^2 + a_1 \theta + a_0)$$



Recap: This is a histogram based non linear intensity modification by a LUT.

Visual Evaluation by Display Presentation

Be aware that every (color) image processing should be done in linear domain of raw camera signals or standardized XYZ/Lab domain. And the visual result is evaluated by viewing at a hopefully calibrated display!

Engineering and Standards

Image Processing Frameworks

- Research: MATLAB, Octave, NumPy, SciPy
- Production: OpenCV, Eigen

GPU Frameworks

- OpenGL
- OpenCL
- Vulkan
- Proprietary: CUDA, Metal

Design Concepts

- Queues, pipeline, interfaces
- (Shared) context
- SIMD
- In/out of place operations

GPU Concepts

- Parallel processing
- Runtime compilation
- Manual resource management for data partitioning

Effort Aspects

- GPU/CPU: Avoid too much copying
- Problem is the availability of certain frameworks
- Cross platform vs. proprietary APIs

Normative and Standards

- Normative: Defined by an independent institute, e.g. ISO/EN/DIN...
- Standards: Company or alliances defined

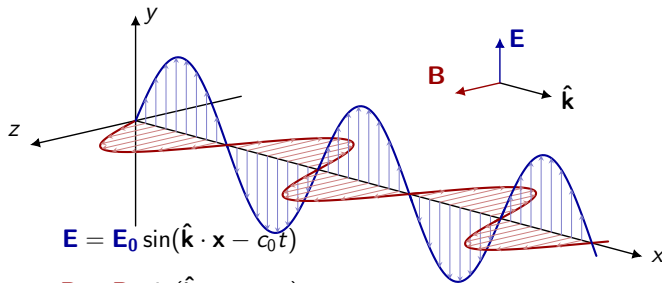
Normative and Standards in Imaging

- Cameras: Genicam (EMVA), EBU Referenz Camera, ACES Referenz Camera...
- Color and Lighting: CIE, ACES...
- Image File Formats: TIFF, jpeg, mpeg...
- Image Processing: **Unfortunately none**
- Output Display: ACES, EBU...

Advanced Topic: Polarization-RGB Cameras

Advanced Topic: Polarization-RGB Cameras

Light as an electromagnetic wave



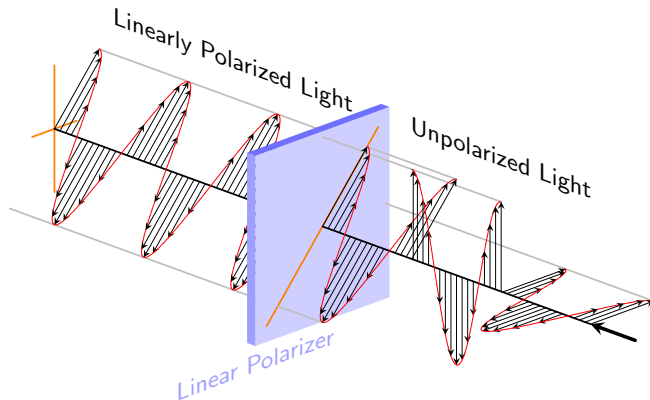
$$\mathbf{E} = \mathbf{E}_0 \sin(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t)$$

$$\mathbf{B} = \mathbf{B}_0 \sin(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t)$$

$$\mathbf{E} \cdot \hat{\mathbf{k}} = 0, \quad \mathbf{B} \cdot \hat{\mathbf{k}} = 0, \quad \mathbf{B} = \frac{1}{c_0} \hat{\mathbf{k}} \times \mathbf{E}$$

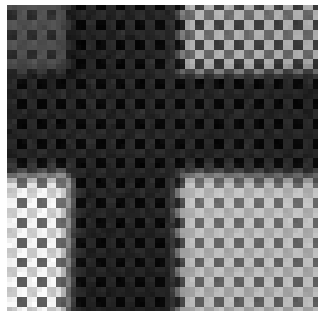
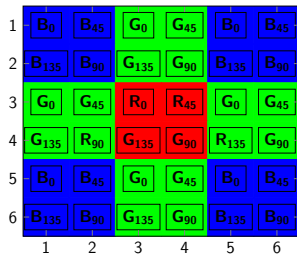
Linear Polarized Light

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The Polarization RGB Sensor

The Pol/RGB sensor has a set of linear polarizers with angles of 0/45/90/135 degrees orientation (left: pixel layout, right: sample region of ColorChecker).



Challenge: Extended Demosaicing

- Problem: Reconstruct much more missing samples, e.g.:
- $\mathbf{d}_{22} = (R_0?, R_{45}?, R_{90}?, R_{135}?, G_0?, G_{45}?, G_{90}?, G_{135}?, B_0?, B_{45}?, B_{90}, B_{135}?)^T$

Polarization Applications

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- Machine Vision:
Photo-elasticity
- Photography/Movie:
Removing reflectance,
more saturated colors

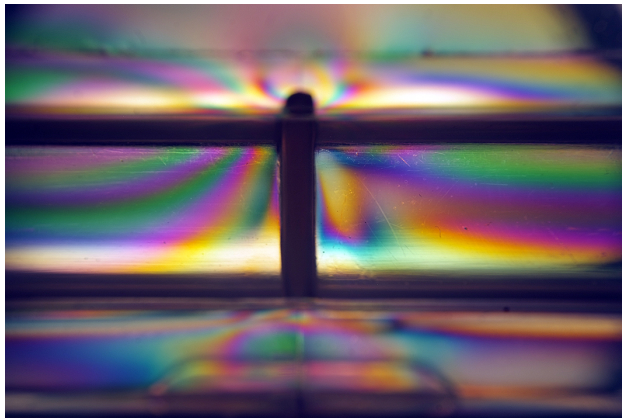


Photo-elasticity Sample

Industrial Applications Examples

Industrial Applications

- Machine Vision: HALCON
- Photography: Photoshop
- Movie: Nuke; DaVinci

... having workflow/pipeline/scripting concepts

A Complete Image Processing Pipeline Example

Small talk about a real world example application:

- Design
- Problems/Pitfalls
- Bottlenecks
- CPU/GPU

QOpenCapture

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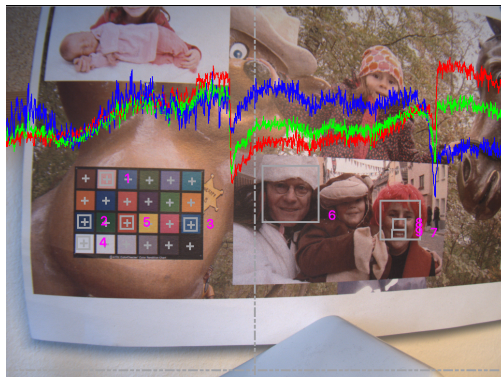


Image Processing Pipeline Input

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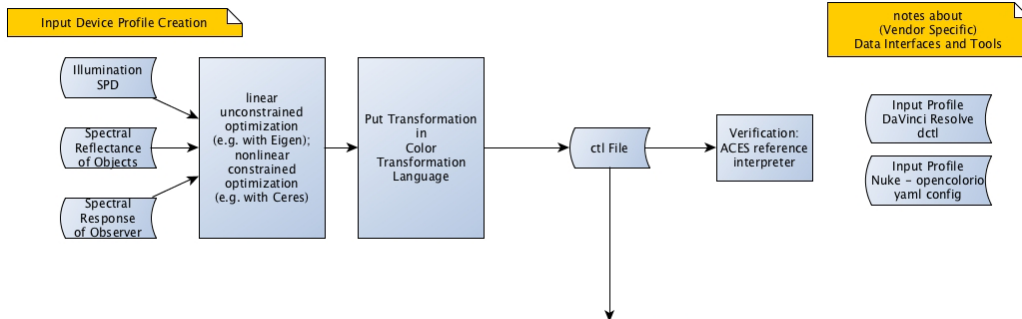


Image Processing Pipeline Processing

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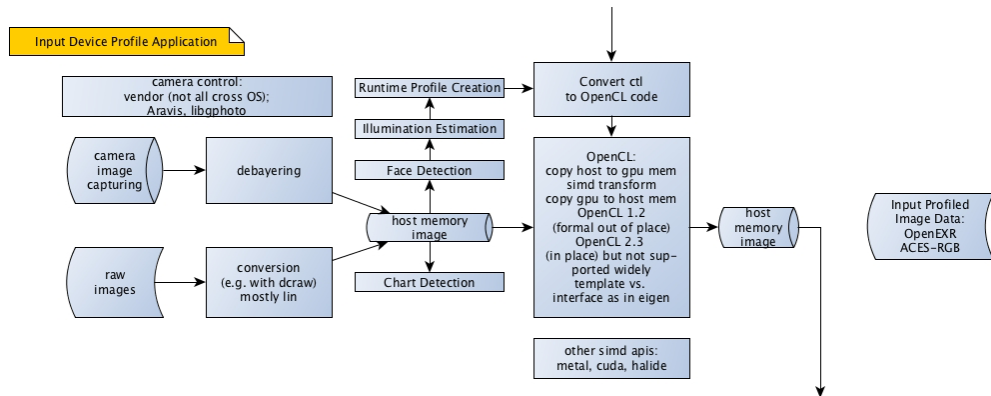
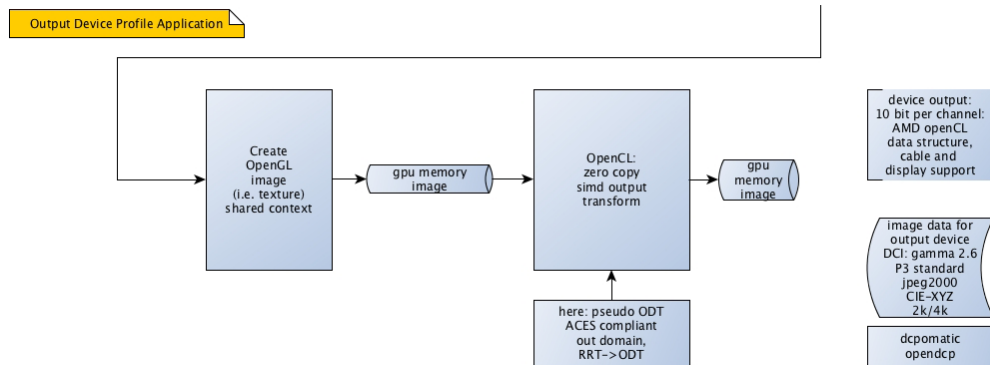


Image Processing Pipeline Output

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Summary

3D Reconstruction, Displaying, Engineering...

- Stereo vision and 3D information from images
- Short glance at:
 - Output display colour processing
 - Engineering, standards
 - Polarization sensors
 - Applications

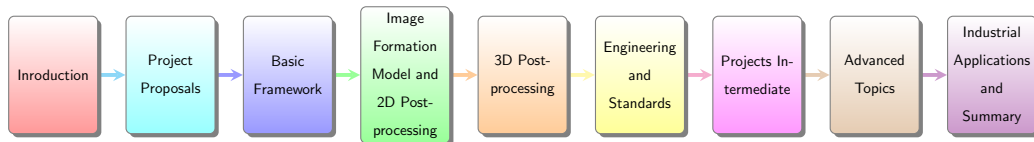
Take Aways

How to get world coordinates from camera pixels locations!

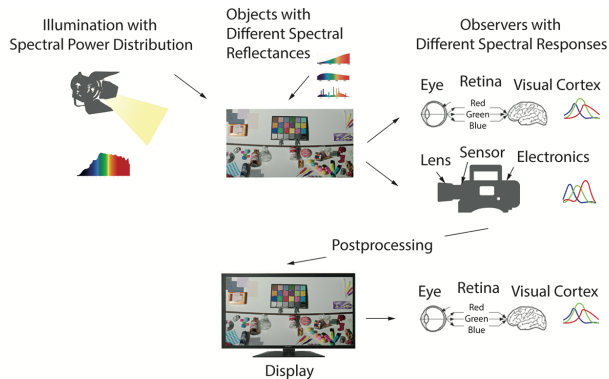
All Lectures Summary

What We Discussed

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Take Away



Vision and Image Processing

The End



- Thank You for Your Attention!
- Any Questions?
- Feedback?

Do not forget: There is a life after Corona!



Marcelo Bertalmio. *Image processing for cinema*. CRC Press, 2014.



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<https://books.google.de/books?id=wcMWpBMMzIkC>.



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